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# Prediction of Mineral Quality of Irrigation Return Flow. Volume IV Data Analysis Utility Programs

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16. ABSTRACT

The development and evaluation of modeling capability to simulate and predict the effects of irrigation on the quality of return flows are documented in the five volumes of this report. The report contains two different modeling packages which represent different levels of detail and sophistication. Volumes I, II and IV pertain to the model package given in Volume III. Volume V contains the more sophisticated model. User's manuals are included in Volumes III and V.

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# PREDICTION OF MINERAL QUALITY OF IRRIGATION RETURN FLOW

## VOLUME IV

## DATA ANALYSIS UTILITY PROGRAMS

by

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EPA-IAG-D4-0371

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# FOREWORD

The Environmental Protection Agency was established to coordinate administration of the major Federal programs designed to protect the quality of our environment.

An important part of the Agency's effort involves the search for information about environmental problems, management techniques and new technologies through which optimum use of the Nation's land and water resources can be assured and the threat pollution poses to the welfare of the American people can be minimized.

EPA's Office of Research and Development conducts this search through a nationwide network of research facilities.

As one of these facilities, the Robert S. Kerr Environmental Research Laboratory is responsible for the management of programs to: (a) investigate the nature, transport, fate and management of pollutants in groundwater; (b) develop and demonstrate methods for treating wastewaters with soil and other natural systems; (c) develop and demonstrate pollution control technologies for irrigation return flows; (d) develop and demonstrate pollution control technologies for animal production wastes; (e) develop and demonstrate technologies to prevent, control or abate pollution from the petroleum refining and petrochemical industries; and (f) develop and demonstrate technologies to manage pollution resulting from combinations of industrial wastewaters or industrial/municipal wastewaters.

This report contributes to the knowledge essential if the EPA is to meet the requirements of environmental laws that it establish and enforce pollution control standards which are reasonable, cost effective and provide adequate protection for the American public.

William C Galegar

William C. Galegar
Director
Robert S. Kerr Environmental
Research Laboratory

# PREFACE

This report is one of a set which documents the development and verification of a digital computer modeling effort to predict the mineral quality changes in return flows occurring as a result of irrigating agricultural lands. The set consists of five separate volumes under one general title as follows:

"Prediction of Mineral Quality of Irrigation Return Flow"

| | |
|---|---|
| Volume I. | Summary Report and Verification |
| Volume II. | Vernal Field Study |
| Volume III. | Simulation Model of Conjunctive Use and Water Quality for a River System or Basin |
| Volume IV. | Data Analysis Utility Programs |
| Volume V. | Detailed Return Flow Salinity and Nutrient Simulation Model |

This set of reports represents the culmination of an effort started in May 1969 by an interagency agreement between the U.S. Bureau of Reclamation and the Federal Water Pollution Control Administration on a joint research proposal on the "Prediction of Mineral Quality of Return Flow Water from Irrigated Land." This research project has had three different project identification numbers during the project period. These numbers (13030 EII, EPA-IAG-048-(D), and EPA-IAG-D4-0371) are given to avoid confusion on the part of individuals who have previously tried to acquire project reports for the earlier project numbers.

# ABSTRACT

This volume of the report contains a description of the data analysis subroutines developed to support the modeling effort described in Volume III. The subroutines were used to evaluate and condition data used in the conjunctive use model. The subroutines include (1) regression analysis, (2) Gaussian probability function, (3) Beta distribution, and (4) Pearson's incomplete gamma function. For each of these subroutines, a brief theory is given plus a program listing and sample problem.

This report was submitted in fulfillment of EPA-IAG-D4-0371 by the Bureau of Reclamation, Engineering and Research Center, under the sponsorship of the Environmental Protection Agency.

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# INTRODUCTION

Hydrology and related fields have a continued requirement for various tools of modern statistical theory. Most of the tools are needed for the examination, extension, and analyses (time series) of different forms of basic or observed data.

Prior to the advent of the electronic computer the extensive use of the techniques of modern statistics was prohibitive because of the many laborious and tedious hand computations. However, as computer technology advanced, resulting in greater computation speeds and larger rapid access storage facilities, the usage of these statistical techniques grew as various forms of computer applications.

It is obvious that no one tool will solve all problems encountered in the analysis of hydrologic data. Further, a comprehensive computer package that was portable and not exclusively propertied was beyond the scope of a reasonable effort both in time and money. Within the limits of a reasonable effort it was determined that a computer package composed of computer applications of regression analyses, tools for the solutions of the normal, beta, and gamma distributions would be a good initial step. Many of the other tools of modern statistical theory such as chi-square tests, Markovian analyses, and pseudo-distribution functions, to name a few, are extensions of the tools contained in this computer package.

In the subsequent narrative a brief description is given of some of the applications of these statistical tools in the field of hydrology.

# REGRESSION ANALYSIS

The computer application of this regression analysis has been designed to be interactive with a terminal. The interactive feature provides easy access to the computer and reduces the narrative required in terms of a user's guide.

Regression analyses are used extensively in the field of hydrology in the manipulation and extension of data. In this respect, the computer application was adapted to fit the more common needs of the practiced hydrologist with some fundamental skill in statistics. This adaptation does not, however, preclude other uses of the regression analysis for problem-solving in areas not related to hydrology.

The subsequent discussion will concern the various features of the computer application and,where necessary,a brief explanation of statistical theory will be included.

## Data Input

The input data are limited to a maximum of 11 variables, one of which is the dependent variable and the remaining 10 are the independent variables. The maximum number of observations (data points) is 500. The two maxima were chosen to fit the majority of hydrology applications.

Each variable must have a title and must also have a reference index of at least one and not more than two digits.

The entry of the above data can be accomplished by any one of two choices. When the sample size is relatively small and the number of variables are few, entry of the data via the terminal is more expedient. Data entered in this manner will be formatted internally and written on an output file labeled OUTO. The internal format used in this case is compatible to that required when data are entered via a read device.

The remaining means of data entry is via a read device. The input file name for the read device is labeled OUTA. Entry of data via the read device is more expedient when the sample size (number of data points) is large, the number of variables many, and where variables are transformed prior to entry into the regression analysis. Surface responses are cases in particular where simple stand-alone mechanisms would be a judicious choice of transformation of variables prior to data entry.

The format required for entry of data concerning the variables is as follows:

Variable title (first line) - 1X,7A10,A9

Variable index and number of data points (second line) - 9X,I3,6X,I3

Data points for variable (third, ...,lines) - 1X,6F13.0/(1X,6F13.0)

Data points must include decimal point in field, if applicable.

The title of the variable is stored in a two-dimensional array labeled LTITLE, the index of the variable is stored in a single-dimensional array labeled MVAR, the data points are stored in a two-dimensional array titled XOB, and the number of data points are stored in a single-dimensional array labeled NPX. A check is made to determine if all variables entered have the same number of data points and when this check fails, a program-controlled error message is displayed at the terminal, otherwise the number of data points entering the regression analysis is stored with the label NP.

All other information pertinent to the regression analysis is entered via the terminal as responses to queries displayed at the terminal. Whenever the query requires a "yes" or "no" response, any single character will suffice for the "no" response; however, the "yes" response must be the three alpha characters YES.

The query format as displayed at the terminal is self-explanatory and the response required is direct, hence any discussion in this respect would be redundant.

## Output Files

The computer application for the regression analysis has provisions for three output files. The file assigned the label OUTPUT is used for all queries displayed at the terminal as well as the display of all error messages displayed at the terminal that are program generated.

The output file assigned the label OUTA is used to store the formatted data, i.e., variable name, index, and data points as accepted via terminal input.

The output file assigned the label OUTB is used to store the results of regression analysis, the intermediate matrices and the residual lists. Options are provided to write the intermediate matrices and residual lists. The intermediate matrices are the (1) raw sums of squares and products (A matrix), (2) covariances (C matrix), and (3) simple coefficients of regression.

The residuals can be listed at each step of the regression analysis. A list of variables from the input set will always be included as part of this output file.

## Program Structure

The program (electronic computer application) structure is composed of the main program and four subroutines and requires approximately $115,000_8$ words to load. The program can be used as a stand-alone application or can be used as five subroutines where four remaining subroutines are monitored by the main subroutine or program.

## Main Program - REGRES

The main program monitors all subroutines and nearly all of the terminal queries are contained in the main program.

### Problem Title

The problem title can be represented with at least one line (79 or less characters per line) or with as many as five lines. The number of lines are stored under the label NTITLE and the lines of title information are stored in a two-dimensional array labeled KTITLE. The lines are entered left justified and will be centered internally on the basis of an 80-character line.

## Variable Sequence

The variable sequence is controlled by the order in which each variable is entered at time of input either via terminal or read device. The variable sequence is stored in a single-dimensional array labeled KRANK. The variable sequence can be changed and only selected variables within the input data set will be allowed to enter the regression analysis. When selected variables are not a part of the input data set, a program-controlled error message will be displayed at the terminal. The index of the dependent variable is stored under the label KVAR.

## Variable Transformation

The purpose of variable transformation is to use a simple linear regression model in terms of the transformed variables rather than a more complicated model in terms of the original variables. When a nonlinear model, defined as nonlinear in parameters to be estimated, can be expressed by adequate transformation of variables in the form of a linear model, it is referred to as being intrinsically linear, and such is the limitation of suitable transforms. This particular computer application of a regression analysis has purposely been further constrained by allowing only six rather simple transformations to be accomplished internally. These six transforms are as follows:

| Form | Alpha configuration |
|---|---|
| $X'_{ij} = \log X_{ij}$ | LOG10 |
| $X'_{ij} = \ln X_{ij}$ | LOGE |
| $X'_{ij} = \sqrt{X_{ij_a}}$ | SQRT |
| $X'_{ij} = X_{ij}$ | EXP |
| $X'_{ij} = \sin X_{ij}$ | SIN |
| $X'_{ij} = \cos X_{ij}$ | COS |

The above transforms are those most commonly used in hydrology where the sine and cosine transforms are a bit far out and the log and ln transforms seem redundant where one can be expressed in terms of the other. In any event, the computer application can be easily modified to discard and replace some of the six transforms listed above or number of transforms performed internally can be expanded to as many as 10.

The alpha configurations of the transforms are stored in a single dimensional array labeled ITRANS.

When it is desirable to represent a "$k^{th}$" order model - single independent variable, another alpha configuration is used. This alpha configuration is the three-character signal POW. The response to this signal will be a query concerning the degree (order) of the polynomial.

The exponents or order of polynomial are stored in the single dimensional array EXPO.

The use of transformed variables should be investigated with an examination of the residuals as obtained as a result of the regression equation. An aid in this examination has been provided by listing the residuals when the dependent variable has been transformed in terms of the original independent variable (exception sine and cosine).

### F Levels

Each of the observations of the dependent variable are random variables, hence all variables that are functions of this set are also random variables. These two functions are the mean square due to the regression and the mean square of the residuals. Both functions have particular distributions, means, variance, and moments. Assume that the errors (deviations) $\varepsilon_i$ are independent, i.e., $N(0,\sigma^2)$ variables, and also that it can be demonstrated that mean square due to the regression multiplied by its degrees of freedom (regression) and the mean square of the residuals multiplied by its degrees of freedom (residual), both will follow a chi-square distribution. It can also be shown that the two variables are independent. Without further discussion it can be said that:

$$F = \frac{\text{mean square due to regression}}{\text{mean square due to residuals}}$$

represents the F ratio and said ratio follows an F distribution.

Nearly all statistical texts or handbooks have tables for the 1 percent (99 percent) and 5 percent (95 percent) F levels.

The regression analysis as represented by this computer application has provision for the entry of F levels. The F level to enter a variable in the regression is stored under the label FLEVEN and the F level to exclude a variable from the regression is stored under the label FLEVRE.

When no F level is given the variables enter the regression in the sequence stored in the array KRANK.

The F level to exclude a variable from the regression should always be less than or equal to the F level to enter a variable in the regression.

It is suggested that one assume two degrees of freedom for the regression and twenty degrees (>20 data points) to estimate F levels. Under such an assumption use F level ∿ 3.5 for 5 percent (95 percent) and F level ∿ 6.0 for 1 percent (99 percent).

## Ranking of Data Points

This application has the provision to rank the data points (observations) by magnitude. In some hydrology studies this sort of license is taken without statistical justification. One would have to possess a keen knowledge of the observed data to use this facet with a linear model. However, the mechanism to do the ranking is a good tool in setting up distribution studies.

The alpha signal to rank in ascending order of magnitude is LOW and to rank in descending order of magnitude the alpha signal is HIGH. These alpha signals are stored in a single dimensional array labeled IRANK.

### Miscellanea

The number of variables entered during input is stored under the label KCONT. The number of variables in regression analysis is stored under the label KNUM. Intermediate storage of ranked data points is provided by the single dimensional array labeled XSORT.

## Subroutine VARTRN

This subroutine is a simple setup mechanism for the six transformations. The argument list is as follows:

KA = Order of variable to be transformed
NUM = Number of data points
KTRAN = Order in the transform list
EXPP = Exponent, if any

All variables are shoved through this subroutine even though no transform is desired. The data points for each variable (transformed or raw) are stored in a two dimensional array labeled XRT. All data ranking, if required, is completed before entry to this subroutine.

## Subroutine PRESRT

This subroutine is used for the sole purpose of setting the indices JB and KB in preparation of the quadratic sort mechanism used in ranking the data points by magnitude.

## Subroutine SORT

This subroutine is used to rank the data points in terms of magnitude. The subroutine uses a quadratic sort mechanism. Detailed discussion of this subroutine is not considered pertinent with respect to the regression analysis.

## Subroutine REGCOR

This subroutine contains the actual computational sequence of the regression model. Before discussion of the details of this subroutine it might be wise to begin with a little of the philosophy of the linear model where the model has the following form:

$$y = a_o + a_1x_1 + a_2X_2 + \ldots + a_nx_n + \varepsilon \quad (0 < n < 11)$$

where: $\varepsilon$ represents the residual and every other term on the right side of the equation represents the regression.

The fundamental notion is to complete the regression of all the variables as a series of straight line regressions where each of the straight line regressions represents a step,or as some say, stage.

The first computational effort is to determine the previously mentioned A matrix which contains the sums and cross products of all the variables in the regression, including the dependent variable.

The second computational effort is to determine the covariances of all the variables, which  is the previously mentioned C matrix.

The third computational effort is to determine the simple correlation coefficients of all the variables, which is the previously mentioned R matrix.

The next and final computational effort prior to the initiation of the straight line regressions or steps is to augment the R matrix in the following manner (n = number of independent variables):

$$B = \begin{bmatrix} R(n \times n) & \vdots & T'(n \times 1) & \vdots & I(n \times n) \\ \cdots\cdots & \cdot & \cdots\cdots\cdots & \cdot & \cdots\cdots\cdots \\ T(1 \times n) & . & S(1 \times 1) & . & 0(1 \times n) \\ \cdots\cdots & \cdot & \cdots\cdots\cdots & \cdot & \cdots\cdots\cdots \\ -I(n \times n) & . & 0(n \times 1) & . & 0(n \times n) \\ \cdots\cdots & \cdot & \cdots\cdots\cdots & \cdot & \cdots\cdots\cdots \end{bmatrix}$$

where: $R(n \times n)$ = matrix of simple correlation coefficients

$T(1 \times n)$ = correlation vector with response to dependent variable

$-I(n \times n)$ = negative identity matrix

$T'(n \times 1)$ = transpose of T

$S(1 \times 1)$ = correlation of response with itself ($R_{yy} \equiv 1$)

$I(n \times n)$ = identity matrix

As an example, consider the following model:

$$y = a_o + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \varepsilon$$

then:

$$B = \left[\begin{array}{cccc|c|cccc}
R_{11} & R_{12} & R_{13} & R_{14} & R_{1y} & 1 & 0 & 0 & 0 \\
R_{21} & R_{22} & R_{23} & R_{24} & R_{2y} & 0 & 1 & 0 & 0 \\
R_{31} & R_{32} & R_{33} & R_{34} & R_{3y} & 0 & 0 & 1 & 0 \\
R_{41} & R_{42} & R_{43} & R_{44} & R_{4y} & 0 & 0 & 0 & 1 \\
\hdashline
R_{y1} & R_{y2} & R_{y3} & R_{y4} & \equiv 1 & 0 & 0 & 0 & 0 \\
\hdashline
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

The B matrix is used to determine the F level of each of the independent variables and choosing that independent variable with the highest F level to enter the regression, if this F level is equal to or greater than the critical F level for entry. The computation of the F level is as follows:

Let: V be a measure of variance

DEGF - degrees of freedom of the residual

then: $V_i = (B_{i,n+1})(B_{i,n+i})/B_{i,i} \quad i = i, \ldots, n$

$$F_i = V_i\ (DEGF)/(B_{n+1,n+1} - V_i) \quad i = 1, \ldots, n$$

Initially the degrees of freedom for the regression equals zero and the degrees of freedom for the residual = NP-1, where NP = number of data points (observations).

The mechanism used in updating the B matrix (augmented R matrix) as each variable enters the regression and the updating of the B matrix after a variable has been excluded from the regression will be discussed in a subsequent portion of this narrative.

Continuing with the philosophy, it might be well to examine a few transformations of variables and the resulting models.

No Transformations (1)

$y = a_o + a_1x + \varepsilon$ - one independent variable

$y = a_o + a_1x_1 + a_2x_2 + \varepsilon$ - two independent variables

$y = a_o + a_1x_1 + a_2x_2 + \ldots + a_nx_n + \varepsilon$ - n independent variables

Logarithmic Transformations (2)

$$\left.\begin{aligned} \ln y &= \ln a_o + a_1 \ln x + \varepsilon \\ \log y &= \log a_o + a_1 \log x + \varepsilon \end{aligned}\right\} \text{one independent variable}$$

$y = a_o + a_1\ln x_1 + a_2\ln x_2 + \varepsilon$ - two independent variables

### Square Root Transformations (3)

$$y = a_o + a_1x_1^{1/2} + a_2x_2^{1/2} + \varepsilon \text{ - two independent variables}$$

### One Independent Variable (4)

$$y = a_o + a_1x + a_2x^2 + \varepsilon \text{ - second-order models}$$

$$y = a_o + a_1x + a_2x^2 + \ldots + a_kx^k + \varepsilon \text{ - } k^{th} \text{ order models}$$

### Two Independent Variables (5)

$$y = a_o + a_1x_1 + a_2x_2 + a_3x_1^2 + a_4x_2^2 + a_4x_1x_2 + \varepsilon \text{ - second-order models - surfaces}$$

$$y = a_o + a_1x + a_2x_2 + a_3x_1^2 + a_4x_1x_2 + a_5x_2^2 + a_6x_1^3 + a_7x_1^2x_2 + a_8x_1x_2^2 + a_9x_2^3 + \varepsilon \text{ - third-order models - surfaces}$$

The above examples of transformations are but a few of the many that are possible. However, the examples should display the general idea of transformations.

Transformations (1), (2), (3), and (4) are handled by the computer application. Transformation (5) can be accomplished by manipulation of the input data file with a stand-alone technique. It can be seen that a third-order model of a surface would be near the limitation of 10 independent variables.

## Algorithms - Updating B Matrix

Let $k$ = index of variable entering regression

$n$ = number of independent variables

$i = 1,2, \ldots, 2n-1$

$j = 1,2, \ldots, 2n-1$

$B_{ij} = D_{kj}/D_{kk} \quad i = k$

$B_{ij} = D_{ij} - D_{ik} \cdot D_{kj}/D_{kk} \quad i \neq k$

Let $k$ = index of variables excluded from regression

$n$ = number of independent variables

$\ell = k + n+1$

$m = 2n-1$

$i = 1,2, \ldots, 2n-1$

$j = 1,2, \ldots, 2n-1$

$B_{ij} = D_{ij}/D_{\ell m} \quad i = k$

$B_{ij} = D_{ij} - D_{im} \cdot D_{mj}/D_{\ell m} \quad i \neq k$

Update $k^{th}$ vector

$i = 1,2, \ldots, 2n-1$

$B_{ik} = D_{ik} - D_{im}/D_{\ell m}$

Note: The D matrix is an exact image of the B matrix from the previous step (stage)

## Labels

| Item | Array |
|---|---|
| A matrix | A(11,11) |
| B matrix | B(21,21) |
| C matrix | C(11,11) |
| D matrix | D(21,21) |
| R matrix | R(11,11) |
| Independent variables | X(11,500) |
| Dependent variables | Y(500) |
| Students T value | STUDT(11) |
| Standard deviation | STDEV(11) |
| Partial F values | PARFL(11) |
| Regression coefficients | REGCOF(10) |
| Variables in regression | KIN(11) |
| F levels | FLEV(11) |
| Variables not in regression | KOUT(11) |
| Original variable index | KSETS(11) |
| Variances | V4(11) |
| Degrees of freedom - regression | NSTEP |
| Degrees of freedom - residual | IDEG,DEGF |
| Step or stage of regression | NSTEP |

All other labels are self-explanatory.

## Statistics

Let $n$ = number of independent variables

$m = n+1$

$B = \left[\quad\right], C = \left[\quad\right]$

Original variance (ORGVAR) independent variable = $C_{mm}$

Percent explained variance (PEREXP) = $(1 - B_{mm})(100)$

Percent unexplained variance (PERUNE) = $B_{mm}(100)$

Multiple correlation coefficient (RMULT) = $\sqrt{1.0 - B_{mm}}$

Residual sum of squares (RESQR) = $C_{mm}.B_{mm}$

Standard error of residuals (STDRES) = $\sqrt{RESQR/DEGF}$

Let $k$ = index of variable

$\ell = k + m$

KJI = number of variables in regression

$i = 1,2, \ldots, KJI$

Regression coefficient (REGCOF(i)) = $B_{im}\sqrt{C_{mm}/C_{ii}}$

Standard error (STDREG(i)) = STDRES $\sqrt{B_{\ell\ell}/C_{ii}}$

Partial F value (PARFL(i)) = DEGF $(B_{im})^2/B_{mm}.B_{\ell\ell}$

The partial F value is tested against critical F level to exclude a variable from the regression because of a nonsignificant contribution to the response of the dependent variable.

## Regression Analysis References

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PROGRAM REGRES     74/74    OPT=1                        FTN 4.2+P380       75/02/28. 13.13.13.       PAGE     1

       PROGRAM REGRES(INPUT,OUTPUT,OUTA,OUTB,OUTD,TAPE5=OUTA,TAPE6=OUTB,
      1TAPE7=OUTD)
       COMMON/VARBLS/ XOB(11,500), XRT(11,500)
       COMMON/TITLES/KTITLE(5,8), LTITLE(11,8),NTITLE
       COMMON/INDICE/KCONT,KNUM,IDY,FLEVEL,NP,CIT,COT,IXR
       COMMON/KEYS/IRANK(11),ITRANS(11),EXPO(11),KSORT(11),KRANK(11)
       DIMENSION MTEST(10) , NPX(11),IFORM(2),ITEM(8),KTEM(8),XSORT(500)
       DIMENSION MVAR(11),NFORM(2)
       INTEGER CIT,COT
       DATA((MTEST(K),K=1,10)=5RLOG10,5RLOGE ,5RSQRT ,5REXP  ,5RSIN  ,
      1  5RCOS  ,  4*5R     )
       DATA(KTEST=3105238)
       CIT=5 $ COT=6 $ IXR=7 $ ISW=0
       PRINT 10
   10  FORMAT(2X*THIS IS A REGRESSION ANALYSIS PACKAGE WITH*
      1  /2X*A MAXIMUM NUMBER OF ELEVEN VARIABLES*)
       ISW=0
       KCONT=0
       DO 20 K=1,11
       MVAR(K)=0
       NPX(K)=0
       IRANK(K)=0
       ITRANS(K)=0
       EXPO(K)=0.0
       KSORT(K)=0
       KRANK(K)=0
   20  CONTINUE
   30  FORMAT(R3)
   35  PRINT 37
   37  FORMAT(2X*DO YOU WANT TO ENTER ANOTHER VARIABLE - YES OR NO*)
       READ 30,ITEST
       IF(ITEST.NE.KTEST) GO TO 100
       PRINT 39
   39  FORMAT(2X*DO YOU WANT TO ENTER VARIABLE VIA TERMINAL-YES OR NO*)
       READ 30,ITEST
       IF(ITEST.EQ.KTEST) GO TO 59
       IF(ISW.NE.1) GO TO 43
       PRINT 41
   41  FORMAT(2X*YOU ARE ATTEMPTING TO READ BEYOND EOF--JOB ABORTED*)
       CALL EXIT
   43  ISW=1
       PRINT 45
   45  FORMAT(2X*YOU WILL READ VARIABLE STRING FROM FILE ON UNIT*
      1  /2X*FIVE-FORMAT(TITLE/INDEX/VARIABLE LIST(1X,6F13.0)*
      2  /2X*DECIMAL POINT MUST BE IN FIELD*)
   46  READ(CIT,49)ITEM
   49  FORMAT(1X,7A10,A9)
       IF(EOF(CIT)) 35,51
   51  READ(CIT,53)INDEX,NA
   53  FORMAT(9X,I3,6X,I3)
       KA=KCONT=KCONT+1 $ NPX(KA)=NA $ MVAR(KA)=INDEX
       IF(KCONT.GT.11) GO TO 1020
       DO 55 K=1,8
       LTITLE(KA,K)=ITEM(K)
   55  CONTINUE
       READ(CIT,57)(XOB(KA,K),K=1,NA)
   57  FORMAT(1X,6F13.0/(1X,6F13.0))
```

```
      PRINT 58,INDEX
   58 FORMAT(2X*INDEX = *I3)
      GO TO 46
   59 PRINT 60
   60 FORMAT(2X*TYPE IN INDEX OF VARIABLE-(1-11)*)
      KCONT=KA=KCONT+1 $ IF(KCONT.GT.11) GO TO 1020
      READ*,INDEX
      PRINT 65
   65 FORMAT(2X*TYPE IN TITLE OF VARIABLE*)
      READ 67,(LTITLE(KA,K), K=1,8)
      WRITE(IXR,49)(LTITLE(KA,K),K=1,8)
   67 FORMAT(7A10,A2)
      MVAR(KA)=INDEX $ JX=0
      PRINT 70
   70 FORMAT(2X*ENTER VARIABLE SET -INCLUDE DECIMAL POINT*/
     1  2X*END WITH A CARRIAGE RETURN*)
   75 READ *, TEMPX
      IF(EOF(5LINPUT)) 95,80
   80 JX=JX+1
      XOB(KA,JX)=TEMPX
      GO TO 75
   85 FORMAT(1X*INDEX = *I3,6X,I3)
   90 FORMAT(1X,6F13.6/(1X,6F13.6))
   95 NPX(KA)=JX
      WRITE(IXR,85)INDEX,JX
      WRITE(IXR,90)(XOB(KA,K),K=1,JX)
      GO TO 35
  100 PRINT 102
  102 FORMAT(2X*ARE ALL VARIABLES ENTERED TO BE IN ANALYSIS WITHOUT*
     1  /2X*TRANSFORMS OR RANKING-YES OR NO*)
      READ 30,ITEST
      IF(ITEST.NE.KTEST) GO TO 104
      DO 103 K=1,KCONT
      KRANK(K)=MVAR(K)
      ITRANS(K)=5ROUT
  103 CONTINUE
      KNUM=KCONT
      GO TO 160
  104 PRINT 105
  105 FORMAT(2X*ENTER INFORMATION FOR REGRESSION ANALYSIS*)
      KNUM=0
  110 PRINT 115
  115 FORMAT(2X*DO YOU WANT TO ENTER MORE INFORMATION-YES OR NO*)
      READ 30,ITEST
      IF(ITEST.NE.KTEST)  GO TO 160
      KNUM=KNUM+1
      PRINT 120
  120 FORMAT(2X*ENTER INDEX OF VARIABLE*)
      READ*,INDEX
      DO 125 K=1,11
      IF(INDEX.NE.MVAR(K)) GO TO 125
      KA=K
      GO TO 135
  125 CONTINUE
      PRINT 130
  130 FORMAT(2X*THIS INDEX IS NOT IN DATA SET--ABORTED--*)
      CALL EXIT
```

```
 135  KRANK(KNUM)=MVAR(KA)
      PRINT 140
 140  FORMAT(2X*ENTER TRANSFORM OR (CR)*)
      READ 145,KTYPE
      IF(EOF(5LINPUT))141,146
 141  KTYPE=5ROUT
      GO TO 146
 145  FORMAT(R5)
 146  ITRANS(KA)=KTYPE $ JTEST=SHIFT(KTYPE,-12) $ JTEST=JTEST.AND.77B
      EXPO(KA)=0.0 $ IF(JTEST.NE.20B) GO TO 149
      PRINT 147
 147  FORMAT(2X*ENTER EXPONENT-DECIMAL POINT INCLUDED*)
      READ*,EXPO(KA)
      GO TO 153
 149  IF(JTEST.NE.27B) GO TO 153
      PRINT 151
 151  FORMAT(2X*ENTER DEGREE OF POLYNOMIAL*)
      READ*,IDEG
      EXPO(KA)=IDEG
 153  PRINT 155
 155  FORMAT(2X*ENTER RANKING INDEX(HIGH,LOW OR (CR)*)
      READ 157,IRANK(KA)
      IF(EOF(5LINPUT))156,158
 156  IRANK(KA)=4R
      GO TO 158
 157  FORMAT(R4)
 158  IF(IRANK(KA).NE.4R     ) KSORT(KA)=1
      GO TO 110
 160  PRINT 165
 165  FORMAT(2X*ENTER INDEX OF DEPENDENT VARIABLE*)
      READ*,ITEST
      DO 167 K=1,11
      IF(ITEST.NE.MVAR(K)) GO TO 167
      KVAR=K
      GO TO 181
 167  CONTINUE
      PRINT 169
 169  FORMAT(2X* DEPENDENT VARIABLE NOT IN DATA SET--ABORTED--*)
      CALL EXIT
 181  DO 185 K=1,KCONT
      IF(KSORT(K).EQ.0) GO TO 185
      DO 182 L=1,500
      XSORT(L)=0.0
 182  CONTINUE
      NS=NPX(K)
      DO 183 L=1,NS
      XSORT(L)=XOB(K,L)
 183  CONTINUE
      CALL PRESRT(XSORT,NS,IRANK(K))
      DO 184 L=1,NS
      XOB(K,L)=XSORT(L)
 184  CONTINUE
 185  CONTINUE
      NP=NPX(1)
      DO 190 K=2,KCONT
      KB=K $ IF(NPX(K).NE.NP) GO TO 200
 190  CONTINUE
```

```
      GO TO 220
  200 PRINT 210,MVAR(KB),NPX(KB)
  210 FORMAT(2X*NO. OF POINTS ARE BAD*/
     1 *VARIABLE INDEX = *I4,2X*NO. OF POINTS = *I4)
      CALL EXIT
  220 MIN=1 $ MAX=KCONT $ MSW=0 $ LINE=0
      IF(MAX.LT.7) GO TO 221
      MSW=1 $ MAX=6
  221 KOT=MAX-MIN+1 $ IFORM(1)=20H
      ENCODE(19,223,IFORM)KOT
      NFORM(1)=IFORM(1) $ NFORM(2)=IFORM(2)
      DO 226 J=1,NP
      IF(LINE.NE.0) GO TO 224
      WRITE(COT,222)
  222 FORMAT(1H1,///2X*LIST OF INPUT VARIABLES*//)
      WRITE(COT,NFORM)(MVAR(JA),JA=MIN,MAX)
  223 FORMAT(4H(1X,I1,14H(8X,*X(*I2*)*))
  224 WRITE(COT,225)(XOB(JB,J),JB=MIN,MAX)
  225 FORMAT(1X,6F13.6)
      LINE=LINE+1 $ IF(LINE.GT.50) LINE=0
  226 CONTINUE
      IF(MSW.EQ.0) GO TO 248
      MIN=7 $ MAX=KCONT $ LINE=0 $ GO TO 221
  248 PRINT 249
  249 FORMAT(2X*DID YOU ENTER VARIABLES IN THE ORDER*/
     1  2X*YOU WOULD LIKE THEM TO ENTER ANALYSIS*/2X*TYPE YES OR NO*)
      READ 30,ITEST
      FLEVEL=99.0
      IF(ITEST.EQ.KTEST) GO TO 270
  250 PRINT 260
  260 FORMAT(2X*TYPE F LEVEL TO ENTER(COMMA) AND F LEVEL TO REMOVE*)
      READ*,FLEVEN,FLEVRE
  270 PRINT 280
  280 FORMAT(2X*ENTER NUMBER OF TITLE CARDS*)
      READ*,NTITLE
      NT = 0
  290 PRINT 300
  300 FORMAT(2X*TYPE IN A TITLE CARD*)
      NT = NT+1
      READ 310, ITEM
  310 FORMAT(8A10)
      IFORM(1)=20H
      LA=8 $ LB=0 $ LC=0
  320 ITEST=ITEM(LA)
  330 JTEST=0 $ JTEST=ITEST.AND.77B
      IF(JTEST.NE.55B) GO TO 350
      ITEST=SHIFT(ITEST,-6) $ LC = LC+ 1
      IF(LC.EQ.10) GO TO 340
      LB=LB+ 1
      GO TO 330
  340 LC=0 $ LA=LA-1 $ IF(LA.LT.1) GO TO 350
      GO TO 320
  350 IF(LB.GT.2) GO TO 370
      DO 360 K=1,8
      KTEM(K)=ITEM(K)
  360 CONTINUE
      GO TO 395
  370  IB=LB/2 $ IBB=LB-IB
       IC=80-LB $ ICC=IC/10 $ ICD=IC-(ICC*10)
       IF(ICD.GT.0) GO TO 380
       ICD=1 $ IBB=IBB-1
  380  ENCODE(20,390,IFORM) IB,ICC,ICD,IBB
  390  FORMAT(*(*I2*X,*I2*A10,A*I2*,*I2*X)*)
       ENCODE(80,IFORM,KTEM)ITEM
  395  DO 400 K=1,8
       KTITLE(NT,K) = KTEM(K)
  400  CONTINUE
       IF(NT.LT.NTITLE) GO TO 290
       DO 430 J=1,KCONT
       KA=J
       DO 420 K=1,10
       KB=K
       IF(ITRANS(J).NE.MTEST(K)) GO TO 420
       CALL VARTRN(KA,NP,KB,EXPO(J))
       GO TO 430
  420  CONTINUE
       KR=0 $ CALL VARTRN(KA,NP,KB,EXPO(J))
  430  CONTINUE
       WRITE(COT,435)
  435  FORMAT(1H1,///1X*PROBLEM INFORMATION*/)
       DO 450 J=1,NTITLE
       WRITE(COT,440)(KTITLE(J,K),K=1,8)
  440  FORMAT(1X,8A10)
  450  CONTINUE
       WRITE(COT,445)
  445  FORMAT(///8X)
       DO 460 K=1,KNUM
       IDV=KRANK(K)
       IF(IDV.EQ.0)  GO TO 460
       WRITE(COT,455)IDV,(LTITLE(IDV,L),L=1,8)
  455  FORMAT(1X*X(*I2*) = *7A10,A2)
  460  CONTINUE
       WRITE(COT,470)NP,KNUM,MVAR(KVAR)
  470  FORMAT(//1X*NUMBER OF EVENTS = *I4,10X*NUMBER OF VARIABLES = *
      1  I4///1X*INDEX OF INDEPENDENT VARIABLE = *I4)
       IF(FLEVEN.EQ.0.0) GO TO 490
       WRITE(COT,480)FLEVEN,FLEVRE
  480  FORMAT(//1X*F LEVEL TO ENTER = *F8.4,5X*F LEVEL TO EXCLUDE = *
      1  F5.4)
       GO TO 510
  490  WRITE(COT,500)
  500  FORMAT(1X*F LEVELS WERE NOT PART OF INPUT AND WILL NOT BE USED*)
  510  CALL REGCOR(KVAR,FLEVEN,FLEVRE,MVAR)
       CALL EXIT
 1000  PRINT 1010
 1010  FORMAT(2X*YOU HAVE READ AN EOF IN ERROR*)
       GO TO 2000
 1020  PRINT 1030
 1030  FORMAT(2X*NUMBER OF VARIABLES GREATER THAN 11*)
 2000  CALL EXIT
       END
```

```
         PROGRAM REGRES     74/74   OPT=1                                  FTN 4.2+P380       75/02/28. 13.13.13.      PAGE     6

      SYMBOLIC REFERENCE MAP (R=1)

 ENTRY POINTS
 12257  REGRES

 VARIABLES      SN  TYPE               RELOCATION
     5  CIT         INTEGER                 INDICE          6  COT       INTEGER                INDICE
    26  EXPO        REAL        ARRAY     KEYS              3  FLEVEL    REAL                   INDICE
 14321  FLEVEN      REAL                                14322  FLEVRE    REAL
 14327  IB          INTEGER                             14330  IBB       INTEGER
 14331  IC          INTEGER                             14332  ICC       INTEGER
 14333  ICD         INTEGER                             14304  IDEG      INTEGER
 14334  IDV         INTEGER                                 2  IDY       INTEGER                INDICE
 14362  IFORM       INTEGER     ARRAY                   14275  INDEX     INTEGER
     0  IRANK       INTEGER     ARRAY     KEYS          14272  ISW       INTEGER
 14364  ITEM        INTEGER     ARRAY                   14274  ITEST     INTEGER
    13  ITRANS      INTEGER     ARRAY     KEYS              7  IXR       INTEGER                INDICE
 14316  J           INTEGER                             14317  JA        INTEGER
 14320  JB          INTEGER                             14303  JTEST     INTEGER
 14309  JX          INTEGER                             14273  K         INTEGER
 14277  KA          INTEGER                             14310  KB        INTEGER
     0  KCONT       INTEGER                 INDICE          1  KNUM      INTEGER                INDICE
 14315  KOT         INTEGER                                54  KRANK     INTEGER     ARRAY      KEYS
    41  KSORT       INTEGER     ARRAY     KEYS          14374  KTEM      INTEGER     ARRAY
 13233  KTEST       INTEGER                                 0  KTITLE    INTEGER     ARRAY      TITLES
 14302  KTYPE       INTEGER                             14305  KVAR      INTEGER
 14306  L           INTEGER                             14324  LA        INTEGER
 14325  LB          INTEGER                             14326  LC        INTEGER
 14314  LINE        INTEGER                                50  LTITLE    INTEGER     ARRAY      TITLES
 14312  MAX         INTEGER                             14311  MIN       INTEGER
 14313  MSW         INTEGER                             14335  MTEST     INTEGER     ARRAY
 15370  MVAR        INTEGER     ARRAY                   14276  NA        INTEGER
 15403  NFORM       INTEGER     ARRAY                       4  NP        INTEGER                INDICE
 14347  NPX         INTEGER     ARRAY                   14307  NS        INTEGER
 14323  NT          INTEGER                               200  NTITLE    INTEGER                TITLES
 14301  TEMPX       REAL                                    0  XOB       REAL        ARRAY      VARBLS
 12574  XRT         REAL        ARRAY     VARBLS        14404  XSORT     REAL        ARRAY

 FILE NAMES        MODE
     0  INPUT      MIXED          4102  OUTA                        6143  OUTB                        10204  OUTD
  2041  OUTPUT     FMT            4102  TAPE5                       6143  TAPE6                       10204  TAPE7

 EXTERNALS         TYPE    ARGS
        EOF        REAL      1                               EXIT                  0
        PRESRT               3                               REGCOR                4
        VARTRN               4

 INLINE FUNCTIONS   TYPE   ARGS
        SHIFT      NO TYPE   2  INTRIN

 STATEMENT LABELS
 13237  10       FMT                          0  20                               13252  30      FMT
 12277  35                                13257  37      FMT                      13276  39      FMT
 13315  41       FMT                      12320  43                               13327  45      FMT
 12323  46                                13354  49      FMT                          0  51         INACTIVE
 13365  53       FMT                          0  55                               13377  57      FMT
 13407  58       FMT                      12363  59                               13415  60      FMT
```

```
         PROGRAM REGRES     74/74   OPT=1                        FTN 4.2+P380      75/02/28. 13.13.13.

STATEMENT LABELS                                                                             PAGE     7
13431  65      FMT                 13453  67      FMT                      13460  70      FMT
12430  75                              0  80          INACTIVE             13476  85      FMT
13502  90      FMT                 12442  95                               12463  100
13524  102     FMT                     0  103                              12502  104
13547  105     FMT                 12505  110                              13560  115     FMT
13577  120     FMT                 12527  125                              13613  130     FMT
12533  135                         13624  140     FMT                          0  141         INACTIVE
13636  145     FMT                 12546  146                              13643  147     FMT
12563  149                         13660  151     FMT                      12573  153
13674  155     FMT                     0  156         INACTIVE             13707  157     FMT
12606  158                         12613  160                              13714  165     FMT
12626  167                         13731  169     FMT                      12632  181
    0  182                             0  183                                  0  184
12657  185                             0  190                              12674  200
13745  210     FMT                 12703  220                              12712  221
13766  222     FMT                 13777  223     FMT                      12736  224
14007  225     FMT                     0  226                              12763  248
14014  249     FMT                     0  250         INACTIVE             14040  260     FMT
12776  270                         14057  280     FMT                      13003  290
14073  300     FMT                 14104  310     FMT                      13014  320
13016  330                         13026  340                              13032  350
    0  360                         13040  370                              13053  380
14116  390     FMT                 13057  395                                  0  400
13102  420                         13111  430                              14134  435     FMT
14147  440     FMT                 14154  445     FMT                          0  450
14165  455     FMT                 13156  460                              14177  470     FMT
14220  480     FMT                 13171  490                              14233  500     FMT
13173  510                             0  1000        INACTIVE             14245  1010    FMT
13201  1020                        14255  1030    FMT                      13203  2000

COMMON BLOCKS   LENGTH
       VARBLS    11000
       TITLES      129
       INDICE        8
       KEYS         55

STATISTICS
  PROGRAM LENGTH              3146B    1638
  BUFFER LENGTH              12245B    5285
  CM LABELED COMMON LENGTH   25670B   11192
```

```
      SUBROUTINE VARTRN     74/74    OPT=1                       FTN 4.2+P380       75/02/28. 13.13.18.
                                                                                          PAGE     1
            SUBROUTINE VARTRN(KA,NUM,KTRAN,EXPP)
            COMMON/VARBLS/ XOB(11,500), XRT(11,500)
            DATA(PI=3.14159265358979)
            LC=KTRAN+1
            GO TO (10,30,40,50,70,90,110,2000,2000,2000,2000),LC
         10 DO 20 K=1,NUM
            XRT(KA,K)=XOB(KA,K)
         20 CONTINUE
            GO TO 2000
         30 DO 35 K=1,NUM
            XRT(KA,K)=0.0 $ IF(XOB(KA,K).GT.0.0) XRT(KA,K)=ALOG10(XOB(KA,K))
         35 CONTINUE
            GO TO 2000
         40 DO 45 K=1,NUM
            XRT(KA,K)=0.0 $ IF(XOB(KA,K).GT.0.0) XRT(KA,K)=ALOG(XOB(KA,K))
         45 CONTINUE
            GO TO 2000
         50 DO 60 K=1,NUM
            XRT(KA,K)=0.0
            IF(XOB(KA,K).GT.0.0) XRT(KA,K)=SQRT(XOB(KA,K))
         60 CONTINUE
            GO TO 2000
         70 DO 80 K=1,NUM
            XRT(KA,K)=0.0
            IF(XOB(KA,K).GT.0.0) XRT(KA,K)=XOB(KA,K)**EXPP
         80 CONTINUE
            GO TO 2000
         90 DO 100 K=1,NUM
            XRT(KA,K)=SIN(XOB(KA,K))
        100 CONTINUE
            GO TO 2000
        110 DO 120 K=1,NUM
            XRT(KA,K)=COS(XOB(KA,K))
        120 CONTINUE
       2000 RETURN
            END
       SUBROUTINE VARTRN     74/74   OPT=1                            FTN 4.2+P380       75/02/28. 13.13.18.
                                                                                                 PAGE     2
       SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
    3  VARTRN

VARIABLES     SN  TYPE           RELOCATION
    0  EXPP      REAL                F.P.          154  K         INTEGER
    0  KA        INTEGER             F.P.            0  KTRAN     INTEGER             F.P.
  153  LC        INTEGER                             0  NUM       INTEGER             F.P.
  152  PI        REAL                                0  XOB       REAL       ARRAY    VARBLS
12574  XRT       REAL       ARRAY    VARBLS

EXTERNALS          TYPE   ARGS
       ALOG      REAL       1 LIBRARY                   ALOG10    REAL       1 LIBRARY
       COS       REAL       1 LIBRARY                   SIN       REAL       1 LIBRARY
       SQRT      REAL       1 LIBRARY

STATEMENT LABELS
   26  10                                  0  20                                      35  30
    0  35                                 53  40                                       0  45
   71  50                                  0  60                                     107  70
    0  80                                126  90                                       0  100
  140  110                                 0  120                                    151  2000

COMMON BLOCKS   LENGTH
       VARBLS    11000

STATISTICS
  PROGRAM LENGTH               155B    109
  CM LABELED COMMON LENGTH   25370B   11000
```

```
SUBROUTINE PRESRT     74/74   OPT=1                          FTN 4.2+P380      75/02/28. 13.13.20.
                                                                                          PAGE     1
                  SUBROUTINE PRESRT(YB,N,KEY)
                  DIMENSION YB (N)
                  COMPA = 0.0
                  COMPB = 0.0
                  COMPC = 0.0
                  COMPA = N
                  COMPB = COMPA * 5.E-1
                  IF ( COMPB . LE. 0.0 )  GO TO 100
                  COMPC = SQRT ( COMPB )
                  JB = COMPC
                  KB = N / JB
                  IF ( MOD (N, KB ) . EQ. 0 )  GO TO 10
                  KB = KB + 1
                  NTOT = KB * JB
                  NP1 = N + 1
            C****  SET KEY FOR RANKING
                  RP=1.E18
                  DO 5 K = NP1 , NTOT
                  YB (K) = RP
                5 CONTINUE
               10 CALL SORT ( YB, JB, KB, YB, N,KEY )
                  RETURN
              100 PRINT 110
              110 FORMAT ( 8X, 39HSORRY NO SORT DUE TO BAD INDICE BREAKUP  )
                  CALL EXIT
                  END
```

```
      SUBROUTINE PRESRT     74/74   OPT=1                             FTN 4.2+P350       75/02/28. 13.13.20.
                                                                                                PAGE     2
      SYMBOLIC REFERENCE MAP (R=1)

  ENTRY POINTS
      3  PRESRT

  VARIABLES      SN  TYPE            RELOCATION
     72  COMPA      REAL                              73  COMPB      REAL
     74  COMPC      REAL                              75  JB         INTEGER
    102  K          INTEGER                           76  KB         INTEGER
      0  KEY        INTEGER            F.P.            0  N          INTEGER             F.P.
    100  NP1        INTEGER                           77  NTOT       INTEGER
    101  RP         REAL                               0  YB         REAL       ARRAY    F.P.

  FILE NAMES        MODE
      OUTPUT    FMT

  EXTERNALS          TYPE   ARGS
      EXIT                    0                          SORT                  6
      SQRT      REAL          1 LIBRARY

  INLINE FUNCTIONS   TYPE   ARGS
      MOD       INTEGER     2  INTRIN

  STATEMENT LABELS
      0  5                             34  10                               44  100
     62  110     FMT

  STATISTICS
    PROGRAM LENGTH              103B      67
```

```
SUBROUTINE SORT       74/74   OPT=1                        FTN 4.2+P380       75/02/28. 13.13.41.      PAGE     1

      SUBROUTINE SORT(ARRAY,JB,KB,Y,LLIM,KEY)
      DIMENSION ARRAY(JB,KB)
      DIMENSION YMIN(200),KPOS(200)
      DIMENSION Y(LLIM),Z(500)
C****  SET KEY FOR RANKING
      ISW=0 $ IF(KEY.EQ.10110710B) ISW=1
C
C
C FILL YMIN WITH THE MINIMUM VALUE FROM EACH COLUMN.  SAVE THE ROW POSIT
      DO 60 K=1,KB
      COLMIN = ARRAY(1,K
      JPOS = 1
      DO 50 J=1,JB
      IF (ARRAY(J,K) .GE. COLMIN) GO TO 50
      COLMIN = ARRAY(J,K)
      JPOS = J
   50 CONTINUE
      YMIN(K) = COLMIN
      KPOS(K) = JPOS
   60 CONTINUE
C
C
C FOR EACH POSITION IN THE ARRAY--
      DO 300 KCOL = 1,KB
      DO 300 KROW = 1,JB
C
C
C FIND MINIMUM VALUE OF YMIN, AND THE COLUMN AND ROW FROM WHICH IT CAME.
      YLIT = YMIN(KCOL)
      LCOL = KCOL
      DO 100 K=KCOL,KB
      IF (YMIN(K) .GE. YLIT) GO TO 100
      YLIT = YMIN(K)
      LCOL = K
  100 CONTINUE
      LROW = KPOS(LCOL)
C
C
C FILL POSITION FROM WHICH IT CAME WITH VALUE IN SLOT WHERE IT IS GOING.
      ARRAY(LROW,LCOL) = ARRAY(KROW,KCOL)
C
C
C STORE THE MINIMUM VALUE OF YMIN IN THE NEXT POSITION IN THE SORTED SEQ
      ARRAY(KROW,KCOL) = YLIT
C
C
C DETERMINE THE NEW MINIMUM OF THE COLUMN FROM WHICH THE LAST MINIMUM WA
C   TAKEN (IF IT IS DIFFERENT FROM THE ONE IN WHICH IT WAS PLACED), AND
C   IT IN THE CORRESPONDING POSITION OF YMIN.
      IF (KCOL .EQ. LCOL) GO TO 200
      COLMIN = ARRAY(1,LCOL)
      JPOS = 1
      DO 150 J=1,JB
      IF (ARRAY(J,LCOL) .GE. COLMIN) GO TO 150
      COLMIN = ARRAY(J,LCOL)
      JPOS = J
  150 CONTINUE
```

```
SUBROUTINE SORT       74/74   OPT=1                      FTN 4.2+P380       75/02/28. 13.13.41.
                                                                                  PAGE     2
            YMIN(LCOL) = COLMIN
            KPOS(LCOL) = JPOS
      C
      C
      C IF THE LAST MINIMUM CAME FROM THE COLUMN INTO WHICH IT WAS SORTED, OR
      C   IF THE LAST SORTED VALUE REPLACED THE MINIMUM OF THE COLUMN IN WHICH
      C   PLACED, FOR THE BALANCE(IF ANY) OF THAT COLUMN FIND THE NEW MINIMUM,
      C   AND PLACE IT IN THE CORRESPONDING POSITION IN YMIN.
        200 IF (KROW .EQ. JB) GO TO 300
            IF (LCOL .EQ. KCOL) GO TO 250
            IF (KROW .NE. KPOS(KCOL)) GO TO 300
        250 J1 = KROW + 1
            COLMIN = ARRAY(J1,KCOL)
            JPOS = J1
            DO 260 J=J1,JB
            IF (ARRAY(J,KCOL) .GE. COLMIN) GO TO 260
            COLMIN = ARRAY(J,KCOL)
            JPOS = J
        260 CONTINUE
            YMIN(KCOL) = COLMIN
            KPOS(KCOL) = JPOS
        300 CONTINUE
            MRS=0
            IF(ISW.EQ.0) GO TO 450
      C****  RESET ARRAY FOR DESCENDING ORDER
            NG=LLIM
            DO 350 K=1,LLIM
            Z(NG)=Y(K)
            NG=NG-1
        350 CONTINUE
            DO 400 K=1,LLIM
            Y(K)=Z(K)
        400 CONTINUE
        450 IF(MRS.NE.0)  GO TO 2000
      C****  LIST THE SORTED ARRAY
            PRINT 500,Y
        500 FORMAT(1X,5F12.4/(1X,5F12.4))
       2000 RETURN
            END
```

```
     SUBROUTINE SORT      74/74    OPT=1                             FTN 4.2+P380      75/02/28. 13.13.41.
                                                                                           PAGE     3
     SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
   3  SORT

VARIABLES     SN  TYPE          RELOCATION
    0  ARRAY      REAL       ARRAY    F.P.         221  COLMIN    REAL
  217  ISW        INTEGER                          223  J         INTEGER
    0  JB         INTEGER             F.P.         222  JPOS      INTEGER
  231  J1         INTEGER                          220  K         INTEGER
    0  KB         INTEGER             F.P.         224  KCOL      INTEGER
    0  KEY        INTEGER             F.P.         544  KPOS      INTEGER    ARRAY
  225  KROW       INTEGER                          227  LCOL      INTEGER
    0  LLIM       INTEGER             F.P.         230  LROW      INTEGER
  232  MRS        INTEGER                          233  NG        INTEGER
    0  Y          REAL       ARRAY    F.P.         226  YLIT      REAL
  234  YMIN       REAL       ARRAY                1054  Z         REAL       ARRAY

FILE NAMES        MODE
     OUTPUT     FMT

STATEMENT LABELS
   41  50                        0  60                      64  100
  110  150                     115  200                    123  250
  141  260                     146  300                      0  350     FMT
    0  400                     173  450                    212  500
  202  2000

STATISTICS
  PROGRAM LENGTH            20708    1080
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```
SUBROUTINE REGCOR     74/74   OPT=1                      FTN 4.2+P380       75/02/28. 13.13.43.      PAGE     1

          SUBROUTINE REGCOR(KVAR,FLEVEN,FLEVRE,MVAR)
          DIMENSION Y(500),SX(11),SY(11),A(11,11), STDERC(11)
          COMMON/VARBLS/XOB(11,500),XRT(11,500)
          COMMON/TITLES/KTITLE(5,8),LTITLE(11,8),NTITLE
          COMMON/INCICE/KCONT,KNUM,IDY,FLEVEL,NP,CIT,COT,IXR
          COMMON/KEYS/IRANK(11),ITRANS(11),EXPO(11),KSORT(11),KRANK(11)
          DIMENSION C(11, 11), CY(11), R(11, 11), RY(11), KFA(11), STUDT(11)
         A  ,FLEV(11),STDEV(11),KOUT(11),PARFL(11)
          DIMENSION SUMS(11),MEANS(11),X(11,500),KSETS(11),IFORM(2),
         1  TEMP(10,10),TEMPA(11),V4(11),KIN(11),B(21.21),D(21,21),
         2  REGCOF(11),STDREG(11),MVAR(11),KEX(11)
          REAL MEANS
          INTEGER CIT, COT
          DATA(IFORM(1)=20H
          DO 1 J=1,11
          KSETS(J)=0
          KEX(J)=0
          DO 1 K=1,500
          X(J,K)=0.0
        1 CONTINUE
          DO 2 K=1,500
          Y(K)=0.0
        2 CONTINUE
          JAY=MVAR(KVAR)
          ISW=0
          FMIN=0
          KMIN=0
          KAX=0
          IMW=0
          NSTEP=0
          PRINT 73
          READ 74,ITEST
          IF(ITEST.EQ.3RYES) ISW=1
          DO 3 K=1,NP
          Y(K)=XRT(JAY,K)
        3 CONTINUF
          NA=0
          YEXP=EXPO(JAY) $ IF(YEXP.EQ.0.0) YEXP=1.0 $ KYTRAN=ITRANS(JAY)
          DO 4 K=1,11
          IF(ITRANS(K).NE.5RPOW      ) GO TO 4
          KA=K  $ GO TO 7
        4 CONTINUE
          DO 6 I=1,11
          IF(KRANK(I).EQ.0)  GO TO 6
          IF(KRANK(I).EQ.JAY) GO TO 6
          J=KRANK(I) $ NA=NA+1 $ KSETS(NA)=J
          DO 5 K=1,NP
          X(NA,K)=XRT(J,K)
        5 CONTINUE
        6 CONTINUE
          GO TO 9
        7 MAX=EXPO(KA) $ KAX=MVAR(KA)
          DO 8 JA=1,MAX
          NA=JA $ KSETS(JA)=NA
          DO 8 JB=1,NP
          X(JA,JB)=XOB(KA,JB)**JA
        8 CONTINUE
```

```
    9 NX=KNUM=NA $ M=0
      IF(NX.LT.1) GO TO 1000
       DO 11 J = 1, 11
       SX(J) = 0.0
       SY(J) = 0.0
       CY(J) = 0.0
       KFA(J) = 0.0
       STUDT(J) = 0.0
       FLEV(J) = 0.0
       STDEV(J) = 0.0
       RY(J) = 0.0
      V4(J)=0.0
      KIN(J)=0
      SUMS (J) = 0.0
      MEANS (J) = 0.0
       DO 11 K = 1, 11
       A(J, K) = 0.0
       C(J, K) = 0.0
       R(J, K) = 0.0
      B(J,K)=0.0
      D(J,K)=0.0
   11   CONTINUE
      DO 12 J = 1, NX
      DO 12 K = 1, NP
      SUMS (J) = SUMS (J) + X(J,K)
   12 CONTINUE
      DENOM = NP
      DO 13 J = 1, NX
      MEANS (J) = SUMS (J) / DENOM
   13 CONTINUE
      SUMY = 0.0
      DO 14 K = 1, NP
      SUMY = SUMY + Y (K)
   14 CONTINUE
      YMEAN = 0.0
      YMEAN = SUMY / DENOM
       MARK = 0
       MT = M
       EXPVAR = 0.0
       IF (M.GT.1) GO TO 15
       MT = NX
       GO TO 15
   15  NQ = NP - 1
C**** WHEN MRS = 1 UNBIASED ESTIMATORS I.E. USE (N-1 )
C**** WHEN MRS.NE. 1 - BIASED ESTIMATORS I.E. USE (N)
      MRS = 1
      BIAS = DENOM
      IF ( MRS. EQ. 1 ) BIAS = DENOM - 1.0
C
C  ******  NO. OF COEFFICIENTS DESIRED CANNOT EXCEED OBSERVATIONS
C
       IF (NQ.LT.MT) MT = NQ
      M1 = MT + 1
      M2 = 2 * MT + 1
       SY2 = 0.0
      MAX=M1+MT
      DO 16 JA=1,MAX
      DO 16 JB=1,MAX
      B(JA,JB)=0.0 $ D(JA,JB)=0.0
  16  CONTINUE
C****  IN MATRIX OPERATION JA=ROW AND JB=COLUMN
      MIN=M1+1
      DO 17 JA=MIN,MAX
      DO 17 JB=1,MT
      JC=JB+M1
      IF(JA.NE.JC) GO TO 17
      B(JA,JB)=-1.0
  17  CONTINUE
      MIN=M1+1
      DO 18 JA=1,MT
      JC=JA+M1
      DO 18 JB=MIN,MAX
      IF(JC.NE.JB)  GO TO 18
      B(JA,JB)=1.0
  18  CONTINUE
C
C  ******  FORM *A* MATRIX FOR INDEPENDENT VARIABLE AND *SY* COLUMN
C  ******  MATRIX FOR DEPENDENT VARIABLE
C  ******  LET  K = ROW  AND  L = COLUMN
C
       DO 35 J = 1, NP
       LA = 1
       MIN = 0
       CONC = 1.0
       CONB = 1.0
       IF (M.EQ.0) CONB = X(LA, J)
       SY2 = SY2 + Y(J) * Y(J)
       DO 35 K = 1, M1
       SY(K) = SY(K) + Y(J) * CONC
       MIN = MIN + 1
       DO 25 L = MIN, M1
       A(K, L) = A(K, L) + CONB * CONC
       A(L, K) = A(K, L)
       IF (M.EQ.0) GO TO 20
       CONC = CONC * X(LA, J)
       GO TO 25
  20   LA = L
       CONB = X(LA, J)
  25   CONTINUE
       IF (M.EQ.0) GO TO 30
       CONB = CONB * X(LA, J)
       CONC = CONB
       GO TO 35
  30   LA = K
       CONC = X(LA, J)
       CONB = CONC
  35   CONTINUE
       A(1, 1) = NP
       NV = 1
C
C  ******  COMPUTATION OF COVARIANCE , INITIAL STANDARD ERROR , AND
C  ******  STANDARD DEVIATION OF DEPENDENT VARIABLE
C
       CYY = SY2 - SY(1) * SY(1) / DENOM
```

```
         STDY = SQRT(CYY / DENOM / (DENOM - 1.0))
         STDEVY = SQRT(CYY / BIAS )
C
C  ******  FORM *C* MATRIX , COVARIANCE , AND *CY* COLUMN MATRIX
C  ******  LET  K = ROW  AND  L = COLUMN
C
  40     DO 45 K = 1, MT
         CY(K) = SY(K + 1) - A(1, K + 1) * SY(1) / DENOM
         DO 45 L = K, MT
         C(K, L) = A(K + 1, L + 1) - A(1, K + 1) * A(1, L + 1) / DENOM
         C(L, K) = C(K, L)
  45     CONTINUE
        DO 50 K=1,M1
        C(K,M1)=CY(X)
  50    CONTINUE
        C(M1,M1)=CYY
         DEGF = NP - NV - 1
C
C  ******  FORM *R* MATRIX , *RY* COLUMN MATRIX , CORRELATION COEFF.
C  ******  LET  K = ROW  AND  L = COLUMN
C
         DO 57 K = 1, MT
        B(K,K)=R(K,K)=1.0
         C2 = C(K, K) * CYY
         IF (C2.LE.0.0) GO TO 1060
C
C  ******  STANDARD DEVIATION OF COMPUTED INDEPENDENT VARIABLE
C****   AND SET B MATRIX ON THE FIRST PASS ONLY
C
         STDEV(K) = SQRT(C(K, K)/ BIAS )
        RY(K)=CY(K)/SQRT(C2) $ IF(MARK.EQ.0)B(K,M1)=RY(K)
         IF (K.EQ.MT) GO TO 57
         MIN = K + 1
         DO 55 L = MIN, MT
         C2 = C(K, K) * C(L, L)
         IF (C2.LE.0.0) GO TO 1060
        R(L,K)=R(K,L)=C(K,L)/SQRT(C2) $ IF(MARK.EQ.0)B(L,K)=B(K,L)=R(K,L)
  55     CONTINUE
  56     KZ = K
  57     CONTINUE
        B(M1,M1)=1.0
        DO 58 K=1,MT
        B(M1,K)=B(K,M1)
  58    CONTINUE
  73    FORMAT(2X*DO YOU WANT TO LIST INTERMEDIATE MATRICES*
     1   /2X*TYPE YES OR NO*)
  74    FORMAT(R3)
        IF(ISW.EQ.0) GO TO 400
C****   LIST A MATRIX - RAW SUM OF SQUARES AND CROSS PRODUCTS
        WRITE(COT,86)
  86    FORMAT(1H1,///1X*RAW SUMS OF SQUARES AND PRODUCTS-OR (A) MATRIX*)
  85    DO 90 LA=1,M1
        TEMPA(LA)=SY(LA)
        DO 90 LB=1,M1
        TEMP(LA,LB)=A(LA,LB)
  90    CONTINUE
        MATE=0
```

```
      ASSIGN 91 TO ISX $ MAX=M1 $ GO TO 300
   91 MAX=M1
      DO 92 J=1,M1
      DO 92 K=1,M1
      TEMP(J,K)=C(J,K)
   92 CONTINUE
C****  LIST COVARIANCE MATRIX
      WRITE(COT,93)
   93 FORMAT( ///1X*COVARIANCE MATRIX OR (C) MATRIX*)
      MATE=1 $ ASSIGN 94 TO ISX $ GO TO 300
C**** LIST R MATRIX SIMPLE COEFFICIENTS OF REGRESSION(INITIAL B MATRIX)
   94 MAX=M1
      WRITE(COT,97)
   97 FORMAT(///1X*R MATRIX SIMPLE COEFFICIENTS OF REGRESSION*
     1  /1X*INITIAL B MATRIX*//)
      DO 98 J=1,MAX
      DO 98 K=1,MAX
      TEMP(J,K)=B(J,K)
   98 CONTINUE
      MATE=1 $ ASSIGN 400 TO ISX $ GO TO 300
  150 PRINT 160
  160 FORMAT(2X*YOUR JOB WENT TO COMPLETION*)
      GO TO 2000
  200 FORMAT(2X*DO YOU WANT RESIDUALS LISTED FOR STEP*I4* -YES OR NO*)
  300 AMAX=-1.0E18
      DO 310 JA=1,MAX
      DO 310 JB=1,MAX
      IF(TEMP(JA,JB).GT.AMAX) AMAX=TEMP(JA,JB)
  310 CONTINUE
      IF(AMAX.GT.1.0)  GO TO 320
      IFO=6RF10.6)  $ GO TO 340
  320 IF(AMAX.GT.1.0E4)  GO TO 330
      IFO=6RF10.3)  $ GO TO 340
  330 IF(AMAX.GT.1.0E6) GO TO 335
      IFO=6RF10.1    $ GO TO 340
  335 IFO=6RE10.4
  340 MIN=1
      DO 360 JA=1,MAX
      IXX=((MIN-1)*10)+1 $ IXY=MAX-MIN+2
      ENCODE(13,350,IFORM)IXX,IXY,IFO
  350 FORMAT(*(*I2*X,*I2,R6)
      IF(MATE.EQ.1) GO TO 355
      WRITE(COT,IFORM)(TEMP(JA,JB),JB=MIN,MAX),TEMPA(JA)
      GO TO 358
  355 WRITE(COT,IFORM)(TEMP(JA,JB),JB=MIN,MAX)
  358 MIN=MIN+1
  360 CONTINUE
      GO TO ISX(91,94,400,700)
C****  INTERCHANGE THE B AND D MATRICES
  400 MAX=M1+MT
      DO 410 JA=1,MAX
      DO 410 JB=1,MAX
      D(JA,JB)=B(JA,JB)
  410 CONTINUE
      IF(IMW.NE.0) GO TO 420
      DO 415 JA=1,MT
      KJO=JA $ KFA(JA)=JA $ KOUT(KJO)=JA
```

```
 415  CONTINUE
 420  KMAX=0 $ VMAX=-1.0E18 $ ITDEG=NP-1-NSTEP $ DEGF=IDEG=ITDEG-1
      DO 430 JA=1,MT
      KN=KOUT(JA) $ IF(KN.LT.0) GO TO 430
      V4(KN)=B(KN,M1)*B(KN,M1)/B(KN,KN)
      FLEV(KN)=(V4(KN)*DEGF)/(B(M1,M1)-V4(KN))
      STUDT(KN)=SQRT(FLEV(KN)) $ IF(V4(KN).LT.VMAX) GO TO 430
      VMAX=V4(KN) $ KMAX=KN
 430  CONTINUE
      FMAX=FLEV(KMAX)
      IF(IMW.NE.0) GO TO 541
      WRITE(COT,435)
 435  FORMAT(1H1,///1X*INITIAL REGRESSION INFORMATION*//)
      IF(KAX.EQ.0) GO TO 440
      WRITE(COT,437)KAX
 437  FORMAT(1X*THIS IS A POWER SERIES ANALYSIS FOR VARIABLE *
     1 I4/1X*VARIABLES AS LISTED ARE DEGREES OF POLYNOMIAL*/)
 440  WRITE(COT,450)
      WRITE(COT,460)(KSETS(K),FLEV(K),STUDT(K),STDEV(K),MEANS(K),K=1,MT)
 450  FORMAT(///1X*VARIABLE*8X*INITIAL*4X*STUDENT'S T*7X*STANDARD*
     1 7X*VARIAELE*/2X*NUMBER*9X*F-LEVEL*21X*DEVIATION*9X*MEAN*/)
 460  FORMAT((4X,I3,2X,4(2X,F13.6)))
      WRITE(COT,470)STDEVY, YMEAN,SUMY
 470  FORMAT(//1X*STANDARD ERROR OF Y = *F13.6,10X*MEAN OF Y = *F13.6
     1  //1X*SUM OF Y = *F13.6)
      IF(FLFVEN.NE.0.0) GO TO 475
      KIN(1)=KN=KFA(1)$FMAX=FLEV(KN) $ KMAX=KN $ GO TO 541
 475  KIN(1)=KN=KMAX
      IF(FMAX.GE.FLEVEN) GO TO 541
      WRITE(COT,500) FMAX
 500  FORMAT(1X*ALL INDEPENDENT VARIABLES FAIL F LEVEL TEST TO*
     1 *ENTER REGRESSION F LEVEL MAX = *F13.6/2X*JOB IS ABORTED*)
      CALL EXIT
 541  IF(IMW.NE.0) GO TO 542
      IMW=1 $ GO TO 400
 542  NSTEP=NSTFP+1 $ IF(NSTEP.GT.MT) GO TO 150
      KJO=0 $ KJI=NSTEP $ IF(FLEVEN.EQ.0.0)KMAX=KFA(NSTEP)
      FMAX=FLEV(KMAX) $ KIN(NSTEP)=KMAX $ IF(KMAX.EQ.KMIN) GO TO 1100
      IF(FLEVEN.EQ.0.0) GO TO 543
      IF(FMAX.LT.FLEVEN) GO TO 1080
 543  DO 545 JA=1,KJI
      KN=KIN(JA)
      DO 544 JB=1,MT
      IF(KOUT(JB).LT.0) GO TO 544
      IF(KN.EQ.JB) KOUT(JB)=-1
 544  CONTINUE
 545  CONTINUE
       DO 547 JA=1,MT
 547  CONTINUE
      KJI=NSTEP
      KN=MJA=MJB=KIN(NSTEP) $ KZ=KN
C****  COMPUTE B MATRIX FOR THIS STEP
 548  MAX=M1+MT
      DO 555 JA=1,MAX
      DO 553 JB=1,MAX
      IF(D(MJA,MJB).EQ.0.0) GO TO 1040
      IF(JA.NE.KN)  GO TO 551
      B(JA,JB)=D(JA,JB)/D(MJA,MJB)
      GO TO 553
551   B(JA,JB)=D(JA,JB)-(D(JA,KZ)*D(KZ,JB)/D(MJA,MJB))
553   CONTINUE
555   CONTINUE
      IF(MJB.NE.MAX) GO TO 558
      DO 556 JA=1,MAX
      IF(JA.EQ.KN) GO TO 556
      B(JA,KN)=D(JA,KN)-(D(JA,MJB)/D(MJA,MJB))
556   CONTINUE
558   ORGVAR=C(M1,M1) $ PEREXP=(1.0-B(M1,M1))*1.0E2
      PERUNE=B(M1,M1)*1.0E2 $ RESQR=C(M1,M1)*B(M1,M1)
      STORES=SQRT(RESQR/DEGF) $ CONST=0.0 $ RMULT=SQRT(1.-B(M1,M1))
      DO 570 JA=1,KJI
      KN=KIN(JA)    $ REGCOF(KN)=B(KN,M1)*SQRT(C(M1,M1)/C(KN,KN))
      MZ=M1+KN
      STOREG(KN)=STORES*SQRT(B(MZ,MZ)/C(KN,KN))
      CONST=CONST+REGCOF(KN)*MEANS(KN)
      PARFL(KN)=(DEGF*B(KN,M1)*B(KN,M1))/(B(M1,M1)*B(MZ,MZ))
570   CONTINUE
      CONST=YMEAN-CONST
      IF(ISW.EQ.0) GO TO 700
700   WRITE(COT,710)NSTEP
710   FORMAT(1H1,////1X*REGRESSION INFORMATION FOR STEP NO. *I4//)
      WRITE(COT,715)KSETS(KMAX),FMAX
715   FORMAT(1X*VARIABLE ENTERING REGRESSION THIS STEP IS X(*
     1  I2*)*5X*F LEVEL TO ENTER =*F13.6/)
      WRITE(COT,720)ORGVAR,PEREXP,PERUNE,RMULT,RESQR,STORES,NSTEP,IDEG,
     1  CONST
720   FORMAT(1X*ORIGINAL VARIANCE*21X,F12.4
     1  /1X*PERCENT EXPLAINED VARIANCE*15X,F9.4
     2  /1X*PERCENT UNEXPLAINED VARIANCE*13X,F9.4
     A  /1X*MULTIPLE CORRELATION COEFF*12X,F12.4
     3  /1X*RESIDUAL SUM OF SQUARES*15X,F12.4
     4  /1X*STANDARD ERROR OF RESIDUALS*11X,F12.4
     5  /1X*DEGREES OF FREEDOM OF REGRESSION*14X,I4
     6  /1X*DEGREES OF FREEDOM OF RESIDUALS*15X,I4
     7  /1X*CONSTANT*30X,F12.4/)
      WRITE(COT,725)
725   FORMAT(1X*VARIABLE*4X*REGRESSION*7X*STANDARD*6X*PARTIAL*
     1  /12X*COEFFICIENTS*8X*ERROR*9X*F VALUE*/)
      DO 727 JA=1,KJI
      KN=KIN(JA) $ KO=KSETS(KN)
      WRITE(COT,726)KO,REGCOF(KN),STOREG(KN),PARFL(KN)
726   FORMAT(3X,I3,4X,F13.6,2(2X,F13.6))
727   CONTINUE
      DIFF2=0.0   $ AEXP=1.0/YEXP
      PRINT 200,NSTEP
      READ 74,ITEST
      IF(ITEST.NE.3RYES) GO TO 932
      LINE=0
      DO 920 J=1,NP
      IF(LINE.NE.0) GO TO 900
      WRITE(COT,860) NSTEP
860   FORMAT(1H1,///1X*LIST OF RESIDUALS FOR STEP NUMBER *I3//)
      DO 880 JA=1,NTITLE
      WRITE(COT,870)(KTITLE(JA,JB),JB=1,8)
```

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 870  FORMAT(1X,7A10,A9)
 880  CONTINUE
      WRITE(COT,890)
 890  FORMAT(//1X*OBS. NO. OBSERVED VALUE  PREDICTED VALUE*
     1  6X*DEVIATION*/)
 900  FOFX=0.0
      DO 903  JB=1,KJI
      KN=KIN(JB) $ FOFX=FOFX+(REGCOF(KN)*X(KN,J))
 903  CONTINUE
      YCOMP=FOFX+CONST $ YOBS=Y(J) $ ITEST=KYTRAN
      IF(ITEST.EQ.5ROUT     ) GO TO 905
      IF(ITEST.EQ.5RSIN     ) GO TO 905
      IF(ITEST.EQ.5RCOS     ) GO TO 905
      YOBS=XOB(JAY,J)
      IF(ITEST.EQ.5RLOG10   ) YCOMP=10.0**YCOMP
      IF(ITEST.EQ.5RLOGE    ) YCOMP=EXP(YCOMP)
      IF(ITEST.EQ.5RSQRT    ) YCOMP=YCOMP*YCOMP
      IF(ITEST.EQ.5REXP     ) YCOMP=YCOMP**AEXP
 905  DIFF=YOBS-YCOMP $ DIFF2=DIFF2+(DIFF*DIFF)
      WRITE(COT,910)J,YOBS,YCOMP,DIFF
 910  FORMAT(1X,I3,5X,F13.6,4X,F13.6,2X,F13.6)
      LINE=LINE+1 $ IF(LINE.GT.50) LINE=0
 920  CONTINUE
      WRITE(COT,930)DIFF2
 930  FORMAT(//1X*SUM OF DEVIATIONS SQUARED = *F20.6)
 932  IF(FLEVRE.EQ.0.0) GO TO 400
      DO 935 JA=1,KJI
      KN=KIN(JA)
      IF(PARFL(KN).LT.FLEVRE) GO TO 940
 935  CONTINUE
      GO TO 400
 940  KMIN=KN $ FMIN=PARFL(KMIN) $ NSTEP=NSTEP-1
      WRITE(COT,945)KSETS(KMIN),FMIN,FLEVRE
 945  FORMAT(//1X*VARIABLE WITH INDEX = *I4* HAS BEEN *
     1  *EXCLUDED FROM REGRESSION WITH F LEVEL = *F15.6
     2  /1X*WHERE F LEVEL TO EXCLUDE = *F15.6)
      JB=0
      DO 950 JA=1,KJI
      KN=KIN(JA)
      IF(KN.EQ.KMIN)  GO TO 950
      JB=JB+1 $ KEX(JB)=KN
 950  CONTINUE
      KJI=KJI-1
      DO 955 JA=1,KJI
      KIN(JA)=KEX(JA)
 955  CONTINUE
      DO 960 JA=1,MT
      KN=KOUT(JA)
      IF(KN.GE.0) GO TO 960
      KOUT(JA)=KMIN
      GO TO 965
 960  CONTINUE
 965  KN=KMIN $ MJA=KN+M1 $ MJB=M1+MT $ KZ=MJB
      ITDEG=NP-1-NSTEP $ DEGF=IDEG=ITDEG
      DO 970 JA=1,MJB
      DO 970 JB=1,MJB
      D(JA,JB)=B(JA,JB)
```

```
  970  CONTINUE
       GO TO 548
 1000  PRINT 1010
 1010  FORMAT(2X*ERROR IN NUMBER OF INDEPENDENT VARIABLES*)
       CALL EXIT
 1040  PRINT 1050
 1050  FORMAT(1X*DIAGONAL ELEMENT IS ZERO-JOB ABORTED*)
       CALL EXIT
 1060  PRINT 1070
 1070  FORMAT(2X*C2 IS LESS THAN OR EQUAL ZERO--ABORTED*)
       CALL EXIT
 1080  WRITE(COT,1090)KSETS(KMAX),FMAX,FLEVEN
 1090  FORMAT(1H1,///1X*VARIABLE WITH INDEX = *I4* HAD A F LEVEL*
      1  * TO ENTER = *F15.6/1X*WHICH IS LESS THAN ENTRY F LEVEL*
      2  * OF *F15.6* ALL OTHER VARIABLES NOT IN REGRESSION ARE*
      3 * IGNORED*//)
       GO TO 150
 1100  WRITE(COT,1110)KSETS(KMAX)
 1110  FORMAT(1H1,///1X*VARIABLE WITH INDEX = *I4* PREVIOUSLY*
      1  * EXCLUCED*)
       GO TO 150
 2000  RETURN
       END
```

```
          SUBROUTINE REGCOR     74/74   OPT=1                         FTN 4.2+P380       75/02/28. 13.13.43.
  CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM                                        PAGE    10
       364     I      700      THIS IF DEGENERATES INTO A SIMPLE TRANSFER TO THE LABEL INDICATED.
```

```
      SUBROUTINE REGCOR     74/74   OPT=1                       FTN 4.2+P380      75/02/28. 13.13.43.      PAGE    11

      SYMBOLIC REFERENCE MAP (R=1)

 ENTRY POINTS
     3  REGCOR

 VARIABLES      SN  TYPE            RELOCATION
  3614  A           REAL       ARRAY                  2574  AEXP      REAL
  2544  AMAX        REAL                             17576  B         REAL       ARRAY
  2520  BIAS        REAL                              4020  C         REAL       ARRAY
     5  CIT         INTEGER          INDICE           2530  CONB      REAL
  2527  CONC        REAL                              2567  CONST     REAL
     6  COT         INTEGER          INDICE           4211  CY        REAL       ARRAY
  2533  CYY         REAL                              2537  C2        REAL
 20467  D           REAL       ARRAY                  2536  DEGF      REAL
  2510  DENOM       REAL                              2601  DIFF      REAL
  2573  DIFF2       REAL                                26  EXPO      REAL       ARRAY    KEYS
  2515  EXPVAR      REAL                              4456  FLEV      REAL       ARRAY
     3  FLEVEL      REAL             INDICE              0  FLEVEN    REAL                F.P.
     0  FLEVRE      REAL              F.P.            2556  FMAX      REAL
  2470  FMIN        REAL                              2576  FOFX      REAL
  2502  I           INTEGER                           2554  IDEG      INTEGER
     2  IDY         INTEGER          INDICE          17367  IFORM     INTEGER    ARRAY
  2545  IFO         INTEGER                           2473  IMH       INTEGER
     0  IRANK       INTEGER    ARRAY KEYS             2467  ISW       INTEGER
  2543  ISX         INTEGER                           2553  ITDEG     INTEGER
  2475  ITEST       INTEGER                             13  ITRANS    INTEGER    ARRAY    KEYS
     7  IXR         INTEGER          INDICE           2546  IXX       INTEGER
  2547  IXY         INTEGER                           2464  J         INTEGER
  2504  JA          INTEGER                           2466  JAY       INTEGER
  2505  JB          INTEGER                           2525  JC        INTEGER
  2465  K           INTEGER                           2501  KA        INTEGER
  2472  KAX         INTEGER                              0  KCONT     INTEGER             INDICE
 21406  KEX         INTEGER    ARRAY                  4430  KFA       INTEGER    ARRAY
 17563  KIN         INTEGER    ARRAY                  2557  KJI       INTEGER
  2550  KJO         INTEGER                           2551  KMAX      INTEGER
  2471  KMIN        INTEGER                           2555  KN        INTEGER
     1  KNUM        INTEGER          INDICE           2572  KO        INTEGER
  4504  KOUT        INTEGER    ARRAY                    54  KRANK     INTEGER    ARRAY    KEYS
 17354  KSETS       INTEGER    ARRAY                    41  KSORT     INTEGER    ARRAY    KEYS
     0  KTITLE      INTEGER    ARRAY TITLES              0  KVAR      INTEGER              F.P.
  2500  KYTRAN      INTEGER                           2540  KZ        INTEGER
  2531  L           INTEGER                           2526  LA        INTEGER
  2541  LB          INTEGER                           2575  LINE      INTEGER
    50  LTITLE      INTEGER    ARRAY TITLES           2507  M         INTEGER
  2513  MARK        INTEGER                           2542  MATE      INTEGER
  2503  MAX         INTEGER                           4545  MEANS     REAL       ARRAY
  2524  MIN         INTEGER                           2560  MJA       INTEGER
  2561  MJB         INTEGER                           2517  MRS       INTEGER
  2514  MT          INTEGER                              0  MVAR      INTEGER    ARRAY     F.P.
  2571  MZ          INTEGER                           2521  M1        INTEGER
  2522  M2          INTEGER                           2476  NA        INTEGER
     4  NP          INTEGER          INDICE           2516  NQ        INTEGER
  2474  NSTEP       INTEGER                            200  NTITLE    INTEGER             TITLES
  2532  NV          INTEGER                           2506  NX        INTEGER
  2562  ORGVAR      REAL                              4517  PARFL     REAL       ARRAY
  2563  PEREXP      REAL                              2564  PERUNE    REAL
  4224  R           REAL       ARRAY                 21360  REGCOF    REAL       ARRAY
```

```
       SUBROUTINE REGCOR     74/74   OPT=1                           FTN 4.2+P380       75/02/28. 13.13.43.      PAGE    12

VARIABLES     SN  TYPE           RELOCATION
 2565  RESQR     REAL                                         2570  RMULT     REAL
 4415  RY        REAL       ARRAY                             4005  STDERC    REAL      *UNDEF
 4471  STDEV     REAL       ARRAY                             2535  STDEVY    REAL
21373  STDREG    REAL       ARRAY                             2566  STDRES    REAL
 2534  STDY      REAL                                         4443  STUDT     REAL       ARRAY
 4532  SUMS      REAL       ARRAY                             2511  SUMY      REAL
 3566  SX        REAL       ARRAY                             3601  SY        REAL       ARRAY
 2523  SY2       REAL                                        17371  TEMP      REAL       ARRAY
17535  TEMPA     REAL       ARRAY                             2552  VMAX      REAL
17550  V4        REAL       ARRAY                             4560  X         REAL       ARRAY
    0  XOB       REAL       ARRAY    VARBLS                  12574  XRT       REAL       ARRAY    VARBLS
 2602  Y         REAL       ARRAY                             2577  YCOMP     REAL
 2477  YEXP      REAL                                         2512  YMEAN     REAL
 2600  YOBS      REAL

FILE NAMES        MODE
       INPUT     FMT                 OUTPUT     FMT

EXTERNALS          TYPE   ARGS
       EXIT                 0                            EXP       REAL       1 LIBRARY
       SQRT      REAL       1 LIBRARY

STATEMFNT LABELS
    0  1                                    0  2                                    0  3
   62  4                                    0  5                                  101  6
  104  7                                    0  8                                  130  9
    0  11                                   0  12                                   0  13
    0  14                                 212  15                                   0  16
  260  17                                 303  18                                 347  20
  353  25                                 364  30                                 371  35
    0  40          INACTIVE                 0  45                                   0  50
    0  55                                   0  56          INACTIVE               523  57
    0  58                                1577  73      FMT                       1607  74      FMT
    0  85          INACTIVE              1614  86      FMT                          0  90
  562  91                                   0  92                                1676  93      FMT
  603  94                                1637  97      FMT                          0  98
  623  150                               1653  160     FMT                       1660  200     FMT
  626  300                                  0  310                                650  320
  654  330                                660  335                                662  340
 1677  350     FMT                        715  355                                733  358
    0  360                                741  400                                  0  410
    0  415                                764  420                               1015  430
 1722  435     FMT                       1734  437     FMT                       1027  440
 1765  450     FMT                       2003  460     FMT                       2015  470     FMT
 1062  475                               2033  500     FMT                       1071  541
 1074  542                               1113  543                               1126  544
    0  545                                  0  547                               1142  548
 1162  551                               1165  553                                  0  555
 1210  556                               1212  558                                  0  570
 1272  700                               2054  710     FMT                       2070  715     FMT
 2116  720     FMT                       2165  725     FMT                       2210  726     FMT
    0  727                               2231  860     FMT                       2246  870     FMT
    0  880                               2254  890     FMT                       1361  900
    0  903                               1427  905                               2273  910     FMT
    0  920                               2303  930     FMT                       1443  932
    0  935                               1457  940                               2317  945     FMT
 1500  950                                  0  955                               1516  960
```

```
    SUBROUTINE REGCOR     74/74   OPT=1                FTN 4.2+P380       75/02/28. 13.13.43.

STATEMENT LABELS                                                             PAGE    13
 1520  965                 0  970                      1543  1000
 2341  1010    FMT      1546  1040    FMT              2352  1050    FMT
 1551  1060             2363  1070    FMT              1554  1080
 2377  1090    FMT      1561  1100                     2427  1110    FMT
 1566  2000

COMMON BLOCKS   LENGTH
    VARBLS       11000
    TITLES         129
    INDICE           8
    KEYS            55

STATISTICS
  PROGRAM LENGTH             21421B    8977
  CM LABELED COMMON LENGTH   25670B   11192
```

```
PROBLEM INFORMATION

                 THIS IS AN EXAMPLE PROBLEM
              DATA FROM RALSTON AND WILF-M.A. EFROYMSON


X( 1) = THIS IS THE VARIABLE NO. 1
X( 2) = THIS IS VARIABLE NO. 2
X( 3) = THIS IS VARIABLE NO. 3
X( 4) = THIS IS VARIABLE NO. 4

NUMBER OF EVENTS =   15          NUMBER OF VARIABLES =    4

INDEX OF INDEPENDENT VARIABLE =    4

F LEVEL TO ENTER =   2.5000     F LEVEL TO EXCLUDE =   2.5000
```

```
LIST OF INPUT VARIABLES

      X( 1)         X( 2)         X( 3)         X( 4)
   32.000000     48.000000     54.000000     15.000000
   36.000000     33.000000     19.000000     16.000000
    3.000000     28.000000     30.000000     14.000000
   12.000000     33.000000     64.000000     22.000000
   36.000000     34.000000     60.000000     24.000000
   24.000000     35.000000     53.000000     19.000000
   19.000000     42.000000     29.000000     13.000000
   20.000000     33.000000     55.000000     15.000000
   27.000000     36.000000     62.000000     23.000000
   15.000000     22.000000     33.000000     12.000000
   45.000000     46.000000     68.000000     25.000000
    9.000000     28.000000     42.000000     17.000000
   11.000000     32.000000     45.000000     18.000000
   33.000000     34.000000     39.000000     19.000000
   21.000000     45.000000     39.000000     18.000000
```

```
RAW SUMS OF SQUARES AND PRODUCTS-OR (A) MATRIX
     15.0     343.0      530.0      692.0      270.0
             9817.0    12793.0    16561.0     6511.0
                       19456.0    24935.0     9657.0
                                  34936.0    13080.0


COVARIANCE MATRIX OR (C) MATRIX
  1973.733   673.667    737.267    337.000
             729.333    484.333    117.000
                       3011.733    624.000
                                   228.000


R MATRIX SIMPLE COEFFICIENTS OF REGRESSION
INITIAL B MATRIX


  1.000000   .561485    .302393    .502364
            1.000000    .326793    .286916
                       1.000000    .753024
                                  1.000000
```

```
INITIAL REGRESSION INFORMATION



VARIABLE        INITIAL    STUDENT'S T        STANDARD        VARIABLE
NUMBER          F-LEVEL                      DEVIATION          MEAN

    1          4.388263       2.094818       11.873540       22.866667
    2          1.166174       1.079895        7.217703       35.333333
    3         17.026242       4.126287       14.667100       46.133333

STANDARD ERROR OF Y =       4.035556          MEAN OF Y =      18.000000

SUM OF Y =    270.000000
```

```
REGRESSION INFORMATION FOR STEP NO.    1

VARIABLE ENTERING REGRESSION THIS STEP IS X( 3)     F LEVEL TO ENTER =    17.026242

ORIGINAL VARIANCE                          228.0000
PERCENT EXPLAINED VARIANCE                  56.7045
PERCENT UNEXPLAINED VARIANCE                43.2955
MULTIPLE CORRELATION COEFF                    .7530
RESIDUAL SUM OF SQUARES                     98.7137
STANDARD ERROR OF RESIDUALS                  2.7556
DEGREES OF FREEDOM OF REGRESSION                  1
DEGREES OF FREEDOM OF RESIDUALS                  13
CONSTANT                                     8.4417

VARIABLE    REGRESSION        STANDARD       PARTIAL
            COEFFICIENTS        ERROR         F VALUE

    3          .207190        .050212       17.026242
```

```
LIST OF RESIDUALS FOR STEP NUMBER    1

                    THIS IS AN EXAMPLE PROBLEM
               DATA FROM RALSTON AND WILF-M.A. EFROYMSON

OBS. NO. OBSERVED VALUE  PREDICTED VALUE       DEVIATION

   1         15.000000         19.629892       -4.629892
   2         16.000000         12.378254        3.621746
   3         14.000000         14.657340        -.657340
   4         22.000000         21.701789         .298211
   5         24.000000         20.873030        3.126970
   6         19.000000         19.422702        -.422702
   7         13.000000         14.450151       -1.450151
   8         15.000000         19.837082       -4.837082
   9         23.000000         21.287409        1.712591
  10         12.000000         15.278909       -3.278909
  11         25.000000         22.530547        2.469453
  12         17.000000         17.143616        -.143616
  13         18.000000         17.765185         .234815
  14         19.000000         16.522047        2.477953
  15         18.000000         16.522047        1.477953

SUM OF DEVIATIONS SQUARED =            98.713653
```

```
REGRESSION INFORMATION FOR STEP NO.    2

VARIABLE ENTERING REGRESSION THIS STEP IS X( 1)     F LEVEL TO ENTER =     2.847233

ORIGINAL VARIANCE                            228.0000
PERCENT EXPLAINED VARIANCE                    65.0072
PERCENT UNEXPLAINED VARIANCE                  34.9928
MULTIPLE CORRELATION COEFF                      .8063
RESIDUAL SUM OF SQUARES                       79.7835
STANDARD ERROR OF RESIDUALS                    2.5785
DEGREES OF FREEDOM OF REGRESSION                    2
DEGREES OF FREEDOM OF RESIDUALS                    12
CONSTANT                                       7.2526

VARIABLE    REGRESSION       STANDARD       PARTIAL
            COEFFICIENTS       ERROR         F VALUE

    3          .182038        .049293      13.638363
    1          .102744        .060890       2.847233
```

```
LIST OF RESIDUALS FOR STEP NUMBER   2

                         THIS IS AN EXAMPLE PROBLEM
                  DATA FROM RALSTON AND WILF-M.A. EFROYMSON

OBS. NO. OBSERVED VALUE  PREDICTED VALUE      DEVIATION

   1          15.000000         20.370429      -5.370429
   2          16.000000         14.410071       1.589929
   3          14.000000         13.021937        .978063
   4          22.000000         20.135929       1.864071
   5          24.000000         21.873634       2.126366
   6          19.000000         19.366438       -.366438
   7          13.000000         14.483803      -1.483803
   8          15.000000         19.319538      -4.319538
   9          23.000000         21.313013       1.686987
  10          12.000000         14.800980      -2.800980
  11          25.000000         24.254635        .745365
  12          17.000000         15.822858       1.177142
  13          18.000000         16.574461       1.425539
  14          19.000000         17.742601       1.257399
  15          18.000000         16.509673       1.490327

SUM OF DEVIATIONS SQUARED =          79.783474
```

```
VARIABLE WITH INDEX =    2 HAD A F LEVEL TO ENTER =          .489651
WHICH IS LESS THAN ENTRY F LEVEL OF        2.500000 ALL OTHER VARIABLES NOT IN REGRESSION ARE IGNORED
```

```
PROBLEM INFORMATION

                       THIS IS AN EXAMPLE PROBLEM
                  DATA FROM DRAPER AND SMITH (HALD DATA)



X( 1) = THIS IS VARIABLE NUMBER ONE
X( 2) = THIS IS VARIABLE NUMBER TWO
X( 3) = THIS IS VARIABLE NUMBER THREE
X( 4) = THIS IS VARIABLE NUMBER FOUR
X( 5) = THIS IS VARIABLE NUMBER FIVE(DEPENDENT VARIABLE)

NUMBER OF EVENTS =   13          NUMBER OF VARIABLES =    5


INDEX OF INDEPENDENT VARIABLE =    5


F LEVEL TO ENTER =   3.9000     F LEVEL TO EXCLUDE =   3.9000
```

```
LIST OF INPUT VARIABLES

      X( 1)        X( 2)        X( 3)        X( 4)        X( 5)
   7.000000    26.000000     6.000000    60.000000    78.500000
   1.000000    29.000000    15.000000    52.000000    74.300000
  11.000000    56.000000     8.000000    20.000000   104.300000
  11.000000    31.000000     8.000000    47.000000    87.600000
   7.000000    52.000000     6.000000    33.000000    95.900000
  11.000000    55.000000     9.000000    22.000000   109.200000
   3.000000    71.000000    17.000000     6.000000   102.700000
   1.000000    31.000000    22.000000    44.000000    72.500000
   2.000000    54.000000    18.000000    22.000000    93.100000
  21.000000    47.000000     4.000000    26.000000   115.900000
   1.000000    40.000000    23.000000    34.000000    83.800000
  11.000000    66.000000     9.000000    12.000000   113.300000
  10.000000    68.000000     8.000000    12.000000   109.400000
```

```
RAW SUMS OF SQUARES AND PRODUCTS-OR (A) MATRIX
      13.0        97.0       626.0       153.0       390.0     1240.5
                1139.0      4922.0       769.0      2620.0    10032.0
                           33050.0      7201.0     15739.0    62027.8
                                        2293.0      4628.0    13981.5
                                                   15062.0    34733.3


COVARIANCE MATRIX OR (C) MATRIX
   415.231     251.077    -372.615    -290.000     775.962
              2905.692    -166.538   -3041.000    2292.954
                           492.308      38.000    -618.231
                                      3362.000   -2481.700
                                                  2715.763


R MATRIX SIMPLE COEFFICIENTS OF REGRESSION
INITIAL B MATRIX


  1.000000     .228579   -.824134   -.245445     .730717
              1.000000   -.139242   -.972955     .816253
                         1.000000    .029537    -.534671
                                    1.000000    -.821305
                                                1.000000
```

```
INITIAL REGRESSION INFORMATION



VARIABLE          INITIAL    STUDENT'S T        STANDARD        VARIABLE
 NUMBER           F-LEVEL                      DEVIATION          MEAN

    1           12.602518       3.550002        5.882394        7.461538
    2           21.960605       4.686214       15.560881       48.153846
    3            4.403417       2.098432        6.405126       11.769231
    4           22.798520       4.774780       16.738180       30.000000

STANDARD ERROR OF Y =      15.043723          MEAN OF Y =      95.423077

SUM OF Y =    12[illegible].500000
```

```
REGRESSION INFORMATION FOR STEP NO.    1

VARIABLE ENTERING REGRESSION THIS STEP IS X( 4)     F LEVEL TO ENTER =     22.798520

ORIGINAL VARIANCE                              2715.7631
PERCENT EXPLAINED VARIANCE                       67.4542
PERCENT UNEXPLAINED VARIANCE                     32.5458
MULTIPLE CORRELATION COEFF                         .8213
RESIDUAL SUM OF SQUARES                         883.8669
STANDARD ERROR OF RESIDUALS                       8.9639
DEGREES OF FREEDOM OF REGRESSION                       1
DEGREES OF FREEDOM OF RESIDUALS                       11
CONSTANT                                        117.5679

VARIABLE    REGRESSION        STANDARD       PARTIAL
           COEFFICIENTS        ERROR          F VALUE

    4         -.738162        .154596       22.798520
```

```
LIST OF RESIDUALS FOR STEP NUMBER    1


                    THIS IS AN EXAMPLE PROBLEM
                DATA FROM DRAPER AND SMITH (HALD DATA)

OBS. NO. OBSERVED VALUE  PREDICTED VALUE      DEVIATION

   1          78.500000        73.278223       5.221777
   2          74.300000        79.183517      -4.883517
   3         104.300000       102.804695       1.495305
   4          87.600000        82.874326       4.725674
   5          95.900000        93.208591       2.691409
   6         109.200000       101.328371       7.871629
   7         102.700000       113.138960     -10.438960
   8          72.500000        85.088812     -12.588812
   9          93.100000       101.328371      -8.228371
  10         115.900000        98.375724      17.524276
  11          83.800000        92.470430      -8.670430
  12         113.300000       108.709989       4.590011
  13         109.400000       108.709989        .690011

SUM OF DEVIATIONS SQUARED =          883.866917
```

```
REGRESSION INFORMATION FOR STEP NO.    2

VARIABLE ENTERING REGRESSION THIS STEP IS X( 1)     F LEVEL TO ENTER =   108.223909

ORIGINAL VARIANCE                          2715.7631
PERCENT EXPLAINED VARIANCE                   97.2471
PERCENT UNEXPLAINED VARIANCE                  2.7529
MULTIPLE CORRELATION COEFF                     .9861
RESIDUAL SUM OF SQUARES                      74.7621
STANDARD ERROR OF RESIDUALS                   2.7343
DEGREES OF FREEDOM OF REGRESSION                   2
DEGREES OF FREEDOM OF RESIDUALS                   10
CONSTANT                                    103.0974

VARIABLE    REGRESSION        STANDARD       PARTIAL
            COEFFICIENTS        ERROR          F VALUE

    4          -.613954        .048645     159.295210
    1          1.439958        .138417     108.223909
```

```
LIST OF RESIDUALS FOR STEP NUMBER   2

                    THIS IS AN EXAMPLE PROBLEM
                 DATA FROM DRAPER AND SMITH (HALD DATA)

OBS. NO. OBSERVED VALUE  PREDICTED VALUE      DEVIATION

   1         78.500000        76.339872        2.160128
   2         74.300000        72.611751        1.688249
   3        104.300000       106.657850       -2.357850
   4         87.600000        90.081102       -2.481102
   5         95.900000        92.916620        2.983380
   6        109.200000       105.429943        3.770057
   7        102.700000       103.733535       -1.033535
   8         72.500000        77.523380       -5.023380
   9         93.100000        92.470318         .629682
  10        115.900000       117.373711       -1.473711
  11         83.800000        83.662917         .137083
  12        113.300000       111.569479        1.730521
  13        109.400000       110.129521        -.729521

SUM OF DEVIATIONS SQUARED =           74.762112
```

```
REGRESSION INFORMATION FOR STEP NO.     3

VARIABLE ENTERING REGRESSION THIS STEP IS X( 2)     F LEVEL TO ENTER =     5.025865

ORIGINAL VARIANCE                          2715.7631
PERCENT EXPLAINED VARIANCE                   98.2335
PERCENT UNEXPLAINED VARIANCE                  1.7665
MULTIPLE CORRELATION COEFF                     .9911
RESIDUAL SUM OF SQUARES                      47.9727
STANDARD ERROR OF RESIDUALS                   2.3087
DEGREES OF FREEDOM OF REGRESSION                   3
DEGREES OF FREEDOM OF RESIDUALS                    9
CONSTANT                                     71.6483

VARIABLE    REGRESSION       STANDARD       PARTIAL
           COEFFICIENTS       ERROR          F VALUE

    4         -.236540        .173288       1.863262
    1         1.451938        .116998     154.007635
    2          .416110        .185610       5.025865
```

```
LIST OF RESIDUALS FOR STEP NUMBER    3

                    THIS IS AN EXAMPLE PROBLEM
                 DATA FROM DRAPER AND SMITH (HALD DATA)

OBS. NO. OBSERVED VALUE  PREDICTED VALUE      DEVIATION

   1          78.500000        78.438314       .061686
   2          74.300000        72.867337      1.432663
   3         104.300000       106.190967     -1.890967
   4          87.600000        89.401637     -1.801637
   5          95.900000        95.643753       .256247
   6         109.200000       105.301777      3.898223
   7         102.700000       104.128673     -1.428673
   8          72.500000        75.591878     -3.091878
   9          93.100000        91.818225      1.281775
  10         115.900000       115.546117       .353883
  11          83.800000        81.702268      2.097732
  12         113.300000       112.244386      1.055614
  13         109.400000       111.624668     -2.224668

SUM OF DEVIATIONS SQUARED =             47.972729

VARIABLE WITH INDEX =    4 HAS BEEN EXCLUDED FROM REGRESSION WITH F LEVEL =        1.863262
WHERE F LEVEL TO EXCLUDE =        3.900000
```

```
REGRESSION INFORMATION FOR STEP NO.    2

VARIABLE ENTERING REGRESSION THIS STEP IS X( 2)     F LEVEL TO ENTER =     5.025865

ORIGINAL VARIANCE                          2715.7631
PERCENT EXPLAINED VARIANCE                   97.8678
PERCENT UNEXPLAINED VARIANCE                  2.1322
MULTIPLE CORRELATION COEFF                     .9893
RESIDUAL SUM OF SQUARES                      57.9045
STANDARD ERROR OF RESIDUALS                   2.4063
DEGREES OF FREEDCM OF REGRESSION                   2
DEGREES OF FREEDOM OF RESIDUALS                   10
CONSTANT                                     52.5773

VARIABLE    REGRESSION       STANDARD       PARTIAL
           COEFFICIENTS        ERROR         F VALUE

    1         1.468306        .121301     146.522655
    2          .662250        .045855     208.581823
```

```
LIST OF RESIDUALS FOR STEP NUMBER   2

                    THIS IS AN EXAMPLE PROBLEM
               DATA FROM DRAPER AND SMITH (HALD DATA)

OBS. NO. OBSERVED VALUE  PREDICTED VALUE      DEVIATION

  1          78.500000         80.074002       -1.574002
  2          74.300000         73.250919        1.049081
  3         104.300000        105.814740       -1.514740
  4          87.600000         89.258477       -1.658477
  5          95.900000         97.292515       -1.392515
  6         109.200000        105.152489        4.047511
  7         102.700000        104.002051       -1.302051
  8          72.500000         74.575420       -2.075420
  9          93.100000         91.275487        1.824513
 10         115.900000        114.537543        1.362457
 11          83.800000         80.535674        3.264326
 12         113.300000        112.437244         .862756
 13         109.400000        112.293440       -2.893440

SUM OF DEVIATIONS SQUARED =           57.904483
```

```
VARIABLE WITH INDEX =    4 PREVIOUSLY EXCLUDED
```

# NORMAL OR GAUSSIAN PROBABILITY FUNCTION

The normal or Gaussian probability function is used extensively in modern statistical theory. Problem-solving in the field of hydrology as well as many other related areas very frequently relies in part on the use of the normal probability function.

In this respect a mechanism is required for the solution of the following integral:

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt = \int_{-\infty}^{x} Z(t)\, dt$$

Several polynomials and rational approximations are available for the solution of the above integral. The error of approximation ($\varepsilon(x)$) is: $|\varepsilon(x)| < 7.5*10^{-8}$ for the better approximations. These polynomials are for the most part approximations of the integral with limits $0 \leq x < \infty$ which utilizes the symmetry of the probability function for the limits $-\infty < x < \infty$.

The subroutine, which is described in the subsequent narrative, is an electronic computer application of the approximation of this same integral with the limits $0 \leq x < \infty$ and again with the advantage of symmetry the integral can be approximated over the range $\infty - < x < \infty$ .

The most commonly used range of the argument "x" in hydrology is $-4.0 \leq x \leq 4.0$, where tails of the probability function, i.e., $> \pm 4.0$, are considered not significantly different than zero or one.

The power series as shown below was used in the computer application to approximate the solution of the aforementioned integral for the limits $-4.0 \leq x \leq 4.0$. The accuracy, or maximum error with this series, is $|\varepsilon(x)| < 1.0 \times 10^{-8}$.

Power series: $$P(x) = 1/2 + Z(x) \sum_{n=0}^{n=50} \frac{x^{2n+1}}{1 \cdot 3 \cdot 5 \ldots (2n+1)} + \varepsilon(x)$$

The series was truncated when $|P(x)_n - P(x)_{n+1}| < 1.0 \times 10^{-9}$.

Another mechanism that is required when the normal probability function is used is that of obtaining the inverse of integral. A popular polynomial approximation to this inverse is as follows:

$$x_p = f(t) = t - \frac{a_o + a_1 t}{1 + b_1 t + b_2 t^2} + \varepsilon(p)$$

where: $0 < p \leq 0.5$ and $t = \sqrt{\ell n \, ^1/_{p^2}}$

and: $a_o = 2.30753$ $a_1 = 0.2760$ |
$b_1 = 0.99229$ $b_2 = 0.0448$ |

and: $|\varepsilon(p)| < 3.0 \times 10^{-3}$

The above polynomial does not provide the same maximum error term as that in the approximation of the integral in the range, $-4.0 \leq x \leq 4.0$. A Taylor series expansion of the higher derivatives was examined as a method of obtaining a better approximation to the inverse.

Let: $Z(x) = \frac{1}{2\pi} \ell^{-x^2/2}$

then: $Z^{(1)}(x) = -xZ(x)$

$Z^{(2)}(x) = (x^2 - 1)\, Z(x)$

$Z^{(3)}(x) = (3x - x^3)\, Z(x)$

$Z^{(4)}(x) = (x^4 - 6x^2 + 3)\, Z(x)$

Note: $d(u,v)/dx = u \frac{dv}{dx} + v \frac{du}{dx}$ was the general form used in obtaining the higher derivatives.

This approach to the approximation became messy in solving for the required root in terms of the fourth degree polynomial resulting from the Taylor series expansion. The technique as used in the computer application for the solution of inverse of the integral combines the polynomial approximation as described earlier and Newton's method of successive approximations.

Let: $x_o = x_p = f(t) + \varepsilon(p)$ (initial guess)

then: $x_1 = x_o - f(x_o)/f'(x_o)$

or: $x_{n+1} = x_n - f(x_n)/f'(x_n)$, n=1, 2, 3, ..., k where: $f(x_n) = P(x_n) - C$

$f'(x_n) = Z(x_n)$

C = True value of the integral

The successive approximations were truncated when $|f(x_n)| < 1.0 \times 10^{-9}$. The maximum error is $|\varepsilon(p)| < 1.0 \times 10^{-8}$ for the range $-4.0 \leq x \leq 4.0$.

The electronic computer application has attached a main program called DRIVER. The main program was used to test the maximum error and can be discarded.

The subroutine used to approximate the solution of the integral and its inverse is titled NORM, where the entry for the inverse is titled NORM1.

The subroutine is entered with three arguments labeled XR, PROB, XC. The label XR is the argument entered for the solution of the integral. The label PROB is the value of the integral. When entry is made via NORM1 (inverse of integral), the label PROB is the true value of the integral and the label XC is the approximation of the argument.

The label SEED is the first guess $(x_o)$ and uses the polynomial approximation shown earlier.

All program-controlled error messages are displayed at the terminal.

```
PROGRAM DRIVER      74/74    OPT=1                            FTN 4.2+P380        75/03/03. 15.11.43.
                                                                                           PAGE     1
        PROGRAM DRIVER(INPUT,OUTPUT,OUT21,TAPE5=OUT21)
        XT=-4.020 $ LINE=0 $ IW=5
     10 XT=XT+0.02  $ IF(XT.GT.4.0) CALL EXIT
        CALL NORM(XT,PROB,XC)
        CALL NORM1(XT,PROB,XC)
        DIFF=XT-XC
        IF(LINE.NE.0) GO TO 35
        WRITE(IW,30)
     30 FORMAT(1H1,////9X*P(X)*3X*X*12X*XC*11X*DIFF*)
     35 WRITE(IW,40)PROB,XT,XC,DIFF
     40 FORMAT(1X,F12.8,3(2X,F12.8))
        LINE =LINE+1 $ IF(LINE.GT.50) LINE=0
        GO TO 10
        END
```

```
          PROGRAM DRIVER     74/74   OPT=1                          FTN 4.2+P380       75/03/03. 15.11.43.
                                                                                                PAGE     2
       SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
 6151  DRIVER

VARIABLES     SN  TYPE           RELOCATION
 6243  DIFF      REAL                              6240  IW        INTEGER
 6237  LINE      INTEGER                           6241  PROB      REAL
 6242  XC        REAL                              6236  XT        REAL

FILE NAMES        MODE
    0  INPUT                2041  OUTPUT                 4102  OUT21                  4102  TAPE5

EXTERNALS          TYPE   ARGS
       EXIT                 0                            NORM                 3
       NORM1                3

STATEMENT LABELS
 6156  10                               6212  30      FMT                      6174  35
 6227  40      FMT

STATISTICS
  PROGRAM LENGTH              101B     65
  BUFFER LENGTH              6143B   3171
```

```
      SUBROUTINE NORM       74/74    OPT=1                          FTN 4.2+P380       75/03/03. 15.11.45.
                                                                                             PAGE     2
      SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
    3  NORM           101  NORM1

VARIABLES     SN  TYPE             RELOCATION
  267  CONST      REAL                                  301  DENOM     REAL
  317  DIFF       REAL                                  275  DZX1      REAL
  276  DZX2       REAL                                  277  DZX3      REAL
  300  DZX4       REAL                                  310  ERROR     REAL
  273  FACT       REAL                                  313  ISIGN     INTEGER
  271  ISX        INTEGER                               304  K         INTEGER
  306  KN         INTEGER                               305  N         INTEGER
  316  NC         INTEGER                               311  P         REAL
  173  FI         REAL                                  302  POFX      REAL
  303  PPOFX      REAL                                    0  PROB      REAL           F.P.
  312  PROD       REAL                                  315  SEED      REAL
  314  TEE        REAL                                  270  X         REAL
    0  XC         REAL               F.P.                 0  XR        REAL           F.P.
  320  X1         REAL                                  272  X2        REAL
  307  ZAP        REAL                                  274  ZX        REAL

FILE NAMES        MODE
      OUTPUT     FMT

EXTERNALS          TYPE    ARGS
      ALOG       REAL       1 LIBRARY                 EXIT                   0
      EXP        REAL       1 LIBRARY                 SQRT      REAL         1 LIBRARY

INLINE FUNCTIONS   TYPE    ARGS
      ABS        REAL       1  INTRIN

STATEMENT LABELS
  177  5       FMT                      21  10                                 22  25
    0  35          INACTIVE              0  40                                215  45       FMT
   71  50                              117  60                                226  80       FMT
  147  90                              151  100                               244  110      FMT
  166  120                             172  150

STATISTICS
  PROGRAM LENGTH              321B     209
```

```
      SUBROUTINE NORM(XR,PROB,XC)
      DATA(PI=3.14159265359)
      CONST=2.0*PI $ CONST=1.0/SQRT(CONST)
      X=ABS(XR) $ IF(X.LE.4.0) GO TO 10
      PRINT 5
    5 FORMAT(1X*ARGUMENT IS GREATER THAN LIMIT VALUE*
     1  * OF INTEGRAL IS SET TO 1.0*)
      PROB=1.0 $ GO TO 150
   10 ASSIGN 150 TO ISX
   25 X2=X*X $ FACT=0.5*X2
      ZX=EXP(-FACT)*CONST $ DZX1=-X*ZX $ DZX2=(X2-1.0)*ZX
      DZX3=((3.0*X)-X2*X)*ZX $ DZX4=((X2*X2)-(6.0*X2)+3.0)*ZX
   35 DENOM=1.0 $ POFX=0.5 $ PPOFX=0.0
      DO 40 K=1,50
      N=K-1 $ KN=2*N+1 $ ZAP=KN $ DENOM=DENOM*ZAP
      POFX=POFX+ZX*X**KN/DENOM
      ERROR=ABS(POFX-PPOFX) $ IF(ERROR.LT.1.0E-9 ) GO TO 50
      PPOFX=POFX
   40 CONTINUE
      PRINT 45,N,ERROR
   45 FORMAT(2X*N = *I4,5X*ERROR = *F20.9)
      CALL EXIT
   50 PROB=POFX $ IF(XR.LT.0.0) PROB=1.0-PROD
      GO TO ISX(150,100)
      ENTRY NORM1
      P=PROD=PROB $ ISIGN=0
      IF(PROD.GE.0.5) GO TO 60
      ISIGN=1 $ PROD=1.0-PROD
   60 IF(P.GT.0.5) P=1.0-P
      IF(P.LE.0.0) P=1.0E-5
      TEE=SQRT(ALOG(1.0/(P*P)))
      SEED=(2.30753+0.27061*TEE)/(1.0+0.99229*TEE+0.04481*TEE*TEE)
      SEED=TEE-SEED $ X=SEED $ NC=0
      IF(SEED.LT.4.2) GO TO 90
      PRINT 80
   80 FORMAT(1X*TRUE VALUE OF INTEGRAL IS GREATER THAN*
     1  /1X*LIMIT ARGUMENT IS SET TO SEED*)
      XC=SEED $ GO TO 120
   90 ASSIGN 100 TO ISX $ GO TO 25
  100 DIFF=POFX-PROD $ IF(ABS(DIFF).LT.1.0E-9) GO TO 120
      X1=X-(DIFF/ZX) $ NC=NC+1
      X=X1 $ IF(NC.LT.30)  GO TO 90
      PRINT 110,X
  110 FORMAT(2X*NO CONVERGENCE FOUND IN X ERROR = *F15.9)
      CALL EXIT
  120 XC=X $ IF(ISIGN.EQ.1) XC=-XC
  150 RETURN
      END
    P(X)            X                XC               DIFF
.00003167     -4.00000000      -4.00000000       -.00000000
.00003446     -3.98000000      -3.98000000       -.00000000
.00003748     -3.96000000      -3.96000000       -.00000000
.00004074     -3.94000000      -3.94000000       -.00000000
.00004427     -3.92000000      -3.92000000       -.00000000
.00004810     -3.90000000      -3.90000000       -.00000000
.00005223     -3.88000000      -3.88000000       -.00000000
.00005669     -3.86000000      -3.86000000       -.00000000
.00006152     -3.84000000      -3.84000000       -.00000000
.00006673     -3.82000000      -3.82000000       -.00000000
.00007235     -3.80000000      -3.80000000       -.00000000
.00007841     -3.78000000      -3.78000000       -.00000000
.00008496     -3.76000000      -3.76000000       -.00000000
.00009201     -3.74000000      -3.74000000       -.00000000
.00009961     -3.72000000      -3.72000000       -.00000000
.00010780     -3.70000000      -3.70000000       -.00000000
.00011662     -3.68000000      -3.68000000       -.00000000
.00012611     -3.66000000      -3.66000000       -.00000000
.00013632     -3.64000000      -3.64000000       -.00000000
.00014730     -3.62000000      -3.62000000       -.00000000
.00015911     -3.60000000      -3.60000000       -.00000000
.00017180     -3.58000000      -3.58000000       -.00000000
.00018543     -3.56000000      -3.56000000       -.00000000
.00020006     -3.54000000      -3.54000000       -.00000000
.00021577     -3.52000000      -3.52000000       -.00000000
.00023263     -3.50000000      -3.50000000       -.00000000
.00025071     -3.48000000      -3.48000000       -.00000000
.00027009     -3.46000000      -3.46000000       -.00000000
.00029086     -3.44000000      -3.44000000       -.00000000
.00031311     -3.42000000      -3.42000000       -.00000000
.00033693     -3.40000000      -3.40000000       -.00000000
.00036243     -3.38000000      -3.38000000       -.00000000
.00038971     -3.36000000      -3.36000000       -.00000000
.00041889     -3.34000000      -3.34000000       -.00000000
.00045009     -3.32000000      -3.32000000       -.00000000
.00048342     -3.30000000      -3.30000000       -.00000000
.00051904     -3.28000000      -3.28000000       -.00000000
.00055706     -3.26000000      -3.26000000       -.00000000
.00059765     -3.24000000      -3.24000000       -.00000000
.00064095     -3.22000000      -3.22000000       -.00000000
.00068714     -3.20000000      -3.20000000       -.00000000
.00073638     -3.18000000      -3.18000000       -.00000000
.00078885     -3.16000000      -3.16000000       -.00000000
.00084474     -3.14000000      -3.14000000       -.00000000
.00090426     -3.12000000      -3.12000000       -.00000000
.00096760     -3.10000000      -3.10000000       -.00000000
.00103500     -3.08000000      -3.08000000       -.00000000
.00110669     -3.06000000      -3.06000000       -.00000000
.00118289     -3.04000000      -3.04000000       -.00000000
.00126387     -3.02000000      -3.02000000       -.00000000
.00134990     -3.00000000      -3.00000000       -.00000000
```

```
       P(X)            X               XC              DIFF
   .00144124     -2.98000000     -2.98000000      -.00000000
   .00153820     -2.96000000     -2.96000000      -.00000000
   .00164106     -2.94000000     -2.94000000      -.00000000
   .00175016     -2.92000000     -2.92000000      -.00000000
   .00186581     -2.90000000     -2.90000000      -.00000000
   .00198838     -2.88000000     -2.88000000      -.00000000
   .00211821     -2.86000000     -2.86000000      -.00000000
   .00225568     -2.84000000     -2.84000000      -.00000000
   .00240118     -2.82000000     -2.82000000      -.00000000
   .00255513     -2.80000000     -2.80000000      -.00000000
   .00271794     -2.78000000     -2.78000000      -.00000000
   .00289007     -2.76000000     -2.76000000      -.00000000
   .00307196     -2.74000000     -2.74000000      -.00000000
   .00326410     -2.72000000     -2.72000000      -.00000000
   .00346697     -2.70000000     -2.70000000      -.00000000
   .00368111     -2.68000000     -2.68000000      -.00000000
   .00390703     -2.66000000     -2.66000000      -.00000000
   .00414530     -2.64000000     -2.64000000      -.00000000
   .00439649     -2.62000000     -2.62000000      -.00000000
   .00466119     -2.60000000     -2.60000000      -.00000000
   .00494002     -2.58000000     -2.58000000      -.00000000
   .00523361     -2.56000000     -2.56000000      -.00000000
   .00554262     -2.54000000     -2.54000000      -.00000000
   .00586774     -2.52000000     -2.52000000      -.00000000
   .00620967     -2.50000000     -2.50000000      -.00000000
   .00656912     -2.48000000     -2.48000000      -.00000000
   .00694685     -2.46000000     -2.46000000      -.00000000
   .00734363     -2.44000000     -2.44000000      -.00000000
   .00776025     -2.42000000     -2.42000000      -.00000000
   .00819754     -2.40000000     -2.40000000      -.00000000
   .00865632     -2.38000000     -2.38000000      -.00000000
   .00913747     -2.36000000     -2.36000000      -.00000000
   .00964187     -2.34000000     -2.34000000      -.00000000
   .01017044     -2.32000000     -2.32000000      -.00000000
   .01072411     -2.30000000     -2.30000000      -.00000000
   .01130384     -2.28000000     -2.28000000      -.00000000
   .01191063     -2.26000000     -2.26000000      -.00000000
   .01254546     -2.24000000     -2.24000000      -.00000000
   .01320938     -2.22000000     -2.22000000      -.00000000
   .01390345     -2.20000000     -2.20000000      -.00000000
   .01462873     -2.18000000     -2.18000000      -.00000000
   .01538633     -2.16000000     -2.16000000      -.00000000
   .01617738     -2.14000000     -2.14000000      -.00000000
   .01700302     -2.12000000     -2.12000000      -.00000000
   .01786442     -2.10000000     -2.10000000      -.00000000
   .01876277     -2.08000000     -2.08000000      -.00000000
   .01969927     -2.06000000     -2.06000000      -.00000000
   .02067516     -2.04000000     -2.04000000      -.00000000
   .02169169     -2.02000000     -2.02000000      -.00000000
   .02275013     -2.00000000     -2.00000000      -.00000000
   .02385176     -1.98000000     -1.98000000      -.00000000
```

```
    P(X)            X               XC             DIFF
.02499790     -1.96000000     -1.96000000      .00000000
.02618984     -1.94000000     -1.94000000      .00000000
.02742895     -1.92000000     -1.92000000      .00000000
.02871656     -1.90000000     -1.90000000     -.00000000
.03005404     -1.88000000     -1.88000000      .00000000
.03144276     -1.86000000     -1.86000000      .00000000
.03288412     -1.84000000     -1.84000000     -.00000000
.03437950     -1.82000000     -1.82000000      .00000000
.03593032     -1.80000000     -1.80000000     -.00000000
.03753798     -1.78000000     -1.78000000     -.00000000
.03920390     -1.76000000     -1.76000000     -.00000000
.04092951     -1.74000000     -1.73999999     -.00000001
.04271622     -1.72000000     -1.72000000      .00000000
.04456546     -1.70000000     -1.70000000      .00000000
.04647866     -1.68000000     -1.68000000      .00000000
.04845723     -1.66000000     -1.66000000     -.00000000
.05050258     -1.64000000     -1.64000000      .00000000
.05261614     -1.62000000     -1.62000000      .00000000
.05479929     -1.60000000     -1.60000000     -.00000000
.05705343     -1.58000000     -1.58000000     -.00000000
.05937994     -1.56000000     -1.56000000     -.00000000
.06178018     -1.54000000     -1.54000000     -.00000000
.06425549     -1.52000000     -1.52000000     -.00000000
.06680720     -1.50000000     -1.50000000     -.00000000
.06943662     -1.48000000     -1.48000000     -.00000000
.07214504     -1.46000000     -1.46000000     -.00000000
.07493370     -1.44000000     -1.44000000     -.00000000
.07780384     -1.42000000     -1.42000000     -.00000000
.08075666     -1.40000000     -1.40000000     -.00000000
.08379332     -1.38000000     -1.38000000     -.00000000
.08691496     -1.36000000     -1.36000000     -.00000000
.09012267     -1.34000000     -1.34000000     -.00000000
.09341751     -1.32000000     -1.32000000     -.00000000
.09680048     -1.30000000     -1.30000000     -.00000000
.10027257     -1.28000000     -1.28000000     -.00000000
.10383469     -1.26000000     -1.26000000     -.00000000
.10748770     -1.24000000     -1.24000000     -.00000000
.11123244     -1.22000000     -1.22000000     -.00000000
.11506967     -1.20000000     -1.20000000     -.00000000
.11900011     -1.18000000     -1.18000000     -.00000000
.12302440     -1.16000000     -1.16000000     -.00000000
.12714315     -1.14000000     -1.14000000     -.00000000
.13135688     -1.12000000     -1.12000000     -.00000000
.13566606     -1.10000000     -1.10000000     -.00000000
.14007109     -1.08000000     -1.08000000     -.00000000
.14457230     -1.06000000     -1.06000000     -.00000000
.14916995     -1.04000000     -1.04000000     -.00000000
.15386423     -1.02000000     -1.02000000     -.00000000
.15865525     -1.00000000     -1.00000000     -.00000000
.16354306      -.98000000      -.98000000     -.00000000
.16852761      -.96000000      -.96000000     -.00000000
```

```
     P(X)            X             XC           DIFF
 .17360878    -.94000000    -.94000000    -.00000000
 .17878638    -.92000000    -.92000000    -.00000000
 .18406013    -.90000000    -.90000000    -.00000000
 .18942965    -.88000000    -.88000000    -.00000000
 .19489452    -.86000000    -.86000000    -.00000000
 .20045419    -.84000000    -.84000000    -.00000000
 .20610805    -.82000000    -.82000000    -.00000000
 .21185540    -.80000000    -.80000000    -.00000000
 .21769544    -.78000000    -.78000000    -.00000000
 .22362729    -.76000000    -.76000000    -.00000000
 .22965000    -.74000000    -.74000000    -.00000000
 .23576250    -.72000000    -.72000000    -.00000000
 .24196365    -.70000000    -.70000000    -.00000000
 .24825223    -.68000000    -.68000000    -.00000000
 .25462691    -.66000000    -.66000000    -.00000000
 .26108630    -.64000000    -.64000000    -.00000000
 .26762889    -.62000000    -.62000000    -.00000000
 .27425312    -.60000000    -.60000000    -.00000000
 .28095731    -.58000000    -.58000000    -.00000000
 .28773972    -.56000000    -.56000000    -.00000000
 .29459852    -.54000000    -.54000000    -.00000000
 .30153179    -.52000000    -.52000000    -.00000000
 .30853754    -.50000000    -.50000000    -.00000000
 .31561370    -.48000000    -.48000000    -.00000000
 .32275811    -.46000000    -.46000000    -.00000000
 .32996855    -.44000000    -.44000000    -.00000000
 .33724273    -.42000000    -.42000000    -.00000000
 .34457826    -.40000000    -.40000000    -.00000000
 .35197271    -.38000000    -.38000000    -.00000000
 .35942357    -.36000000    -.36000000    -.00000000
 .36692826    -.34000000    -.34000000    -.00000000
 .37448417    -.32000000    -.32000000    -.00000000
 .38208858    -.30000000    -.30000000    -.00000000
 .38973875    -.28000000    -.28000000    -.00000000
 .39743189    -.26000000    -.26000000    -.00000000
 .40516513    -.24000000    -.24000000    -.00000000
 .41293558    -.22000000    -.22000000    -.00000000
 .42074029    -.20000000    -.20000000    -.00000000
 .42857628    -.18000000    -.18000000    0.00000000
 .43644054    -.16000000    -.16000000     .00000000
 .44433000    -.14000000    -.14000000    -.00000000
 .45224157    -.12000000    -.12000000     .00000000
 .46017216    -.10000000    -.10000000    -.00000000
 .46811863    -.08000000    -.08000000     .00000000
 .47607782    -.06000000    -.06000000    -.00000000
 .48404656    -.04000000    -.04000000     .00000000
 .49202169    -.02000000    -.02000000    -.00000000
 .50000000     .00000000     .00000000    -.00000000
 .50797831     .02000000     .02000000     .00000000
 .51595344     .04000000     .04000000    -.00000000
 .52392218     .06000000     .06000000    -.00000000
```

```
      P(X)             X              XC            DIFF
  .53188137      .08000000      .08000000      .00000000
  .53982784      .10000000      .10000000      .00000000
  .54775843      .12000000      .12000000      .00000000
  .55567000      .14000000      .14000000      .00000000
  .56355946      .16000000      .16000000      .00000000
  .57142372      .18000000      .18000000      .00000000
  .57925971      .20000000      .20000000      .00000000
  .58706442      .22000000      .22000000      .00000000
  .59483487      .24000000      .24000000      .00000000
  .60256811      .26000000      .26000000      .00000000
  .61026125      .28000000      .28000000      .00000000
  .61791142      .30000000      .30000000      .00000000
  .62551583      .32000000      .32000000      .00000000
  .63307174      .34000000      .34000000      .00000000
  .64057643      .36000000      .36000000      .00000000
  .64802729      .38000000      .38000000      .00000000
  .65542174      .40000000      .40000000      .00000000
  .66275727      .42000000      .42000000      .00000000
  .67003145      .44000000      .44000000      .00000000
  .67724189      .46000000      .46000000      .00000000
  .68438630      .48000000      .48000000      .00000000
  .69146246      .50000000      .50000000      .00000000
  .69846821      .52000000      .52000000      .00000000
  .70540148      .54000000      .54000000      .00000000
  .71226028      .56000000      .56000000      .00000000
  .71904269      .58000000      .58000000      .00000000
  .72574688      .60000000      .60000000      .00000000
  .73237111      .62000000      .62000000      .00000000
  .73891370      .64000000      .64000000      .00000000
  .74537309      .66000000      .66000000      .00000000
  .75174777      .68000000      .68000000      .00000000
  .75803635      .70000000      .70000000      .00000000
  .76423750      .72000000      .72000000      .00000000
  .77035000      .74000000      .74000000      .00000000
  .77637271      .76000000      .76000000      .00000000
  .78230456      .78000000      .78000000      .00000000
  .78814460      .80000000      .80000000      .00000000
  .79389195      .82000000      .82000000      .00000000
  .79954581      .84000000      .84000000      .00000000
  .80510548      .86000000      .86000000      .00000000
  .81057035      .88000000      .88000000      .00000000
  .81593987      .90000000      .90000000      .00000000
  .82121362      .92000000      .92000000      .00000000
  .82639122      .94000000      .94000000      .00000000
  .83147239      .96000000      .96000000      .00000000
  .83645694      .98000000      .98000000      .00000000
  .84134475     1.00000000     1.00000000      .00000000
  .84613577     1.02000000     1.02000000      .00000000
  .85083005     1.04000000     1.04000000      .00000000
  .85542770     1.06000000     1.06000000      .00000000
  .85992891     1.08000000     1.08000000      .00000000
```

```
     P(X)           X              XC             DIFF
.86433394      1.10000000     1.10000000      .00000000
.86864312      1.12000000     1.12000000      .00000000
.87285685      1.14000000     1.14000000      .00000000
.87697560      1.16000000     1.16000000      .00000000
.88099989      1.18000000     1.18000000      .00000000
.88493033      1.20000000     1.20000000      .00000000
.88876756      1.22000000     1.22000000      .00000000
.89251230      1.24000000     1.24000000      .00000000
.89616532      1.26000000     1.26000000      .00000000
.89972743      1.28000000     1.28000000      .00000000
.90319952      1.30000000     1.30000000      .00000000
.90658249      1.32000000     1.32000000      .00000000
.90987733      1.34000000     1.34000000      .00000000
.91308504      1.36000000     1.36000000      .00000000
.91620668      1.38000000     1.38000000      .00000000
.91924334      1.40000000     1.40000000      .00000000
.92219616      1.42000000     1.42000000      .00000000
.92506630      1.44000000     1.44000000      .00000000
.92785496      1.46000000     1.46000000      .00000000
.93056338      1.48000000     1.48000000      .00000000
.93319280      1.50000000     1.50000000      .00000000
.93574451      1.52000000     1.52000000      .00000000
.93821982      1.54000000     1.54000000      .00000000
.94062006      1.56000000     1.56000000      .00000000
.94294657      1.58000000     1.58000000      .00000000
.94520071      1.60000000     1.60000000      .00000000
.94738386      1.62000000     1.62000000      .00000000
.94949742      1.64000000     1.64000000      .00000000
.95154277      1.66000000     1.66000000      .00000000
.95352134      1.68000000     1.68000000      .00000000
.95543454      1.70000000     1.70000000      .00000000
.95728378      1.72000000     1.72000000      .00000000
.95907049      1.74000000     1.73999999      .00000001
.96079610      1.76000000     1.76000000      .00000000
.96246202      1.78000000     1.78000000      .00000000
.96406968      1.80000000     1.80000000      .00000000
.96562050      1.82000000     1.82000000      .00000000
.96711588      1.84000000     1.84000000      .00000000
.96855724      1.86000000     1.86000000      .00000000
.96994596      1.88000000     1.88000000      .00000000
.97128344      1.90000000     1.90000000      .00000000
.97257105      1.92000000     1.92000000      .00000000
.97381016      1.94000000     1.94000000      .00000000
.97500210      1.96000000     1.96000000      .00000000
.97614824      1.98000000     1.98000000      .00000000
.97724987      2.00000000     2.00000000      .00000000
.97830831      2.02000000     2.02000000      .00000000
.97932484      2.04000000     2.04000000      .00000000
.98030073      2.06000000     2.06000000      .00000000
.98123723      2.08000000     2.08000000      .00000000
.98213558      2.10000000     2.10000000      .00000000
```

```
      P(X)            X               XC              DIFF
  .98299698     2.12000000      2.12000000      .00000000
  .98382262     2.14000000      2.14000000      .00000000
  .98461367     2.16000000      2.16000000      .00000000
  .98537127     2.18000000      2.18000000      .00000000
  .98609655     2.20000000      2.20000000      .00000000
  .98679062     2.22000000      2.22000000      .00000000
  .98745454     2.24000000      2.24000000      .00000000
  .98808937     2.26000000      2.26000000      .00000000
  .98869616     2.28000000      2.28000000      .00000000
  .98927589     2.30000000      2.30000000      .00000000
  .98982956     2.32000000      2.32000000      .00000000
  .99035813     2.34000000      2.34000000      .00000000
  .99086253     2.36000000      2.36000000      .00000000
  .99134368     2.38000000      2.38000000      .00000000
  .99180246     2.40000000      2.40000000      .00000000
  .99223975     2.42000000      2.42000000      .00000000
  .99265637     2.44000000      2.44000000      .00000000
  .99305315     2.46000000      2.46000000      .00000000
  .99343088     2.48000000      2.48000000      .00000000
  .99379033     2.50000000      2.50000000      .00000000
  .99413226     2.52000000      2.52000000      .00000000
  .99445738     2.54000000      2.54000000      .00000000
  .99476639     2.56000000      2.56000000      .00000000
  .99505998     2.58000000      2.58000000      .00000000
  .99533881     2.60000000      2.60000000      .00000000
  .99560351     2.62000000      2.62000000      .00000000
  .99585470     2.64000000      2.64000000      .00000000
  .99609297     2.66000000      2.66000000      .00000000
  .99631889     2.68000000      2.68000000      .00000000
  .99653303     2.70000000      2.70000000      .00000000
  .99673590     2.72000000      2.72000000      .00000000
  .99692804     2.74000000      2.74000000      .00000000
  .99710993     2.76000000      2.76000000      .00000000
  .99728206     2.78000000      2.78000000      .00000000
  .99744487     2.80000000      2.80000000      .00000000
  .99759882     2.82000000      2.82000000      .00000000
  .99774432     2.84000000      2.84000000      .00000000
  .99788179     2.86000000      2.86000000      .00000000
  .99801162     2.88000000      2.88000000      .00000000
  .99813419     2.90000000      2.90000000      .00000000
  .99824984     2.92000000      2.92000000      .00000000
  .99835894     2.94000000      2.94000000      .00000000
  .99846180     2.96000000      2.96000000      .00000000
  .99855876     2.98000000      2.98000000      .00000000
  .99865010     3.00000000      3.00000000      .00000000
  .99873613     3.02000000      3.02000000      .00000000
  .99881711     3.04000000      3.04000000      .00000000
  .99889331     3.06000000      3.06000000      .00000000
  .99896500     3.08000000      3.08000000      .00000000
  .99903240     3.10000000      3.10000000      .00000000
  .99909574     3.12000000      3.12000000      .00000000
```

```
    P(X)            X              XC             DIFF
.99915526      3.14000000     3.14000000      .00000000
.99921115      3.16000000     3.16000000      .00000000
.99926362      3.18000000     3.18000000      .00000000
.99931286      3.20000000     3.20000000      .00000000
.99935905      3.22000000     3.22000000      .00000000
.99940235      3.24000000     3.24000000      .00000000
.99944294      3.26000000     3.26000000      .00000000
.99948096      3.28000000     3.28000000      .00000000
.99951658      3.30000000     3.30000000      .00000000
.99954991      3.32000000     3.32000000      .00000000
.99958111      3.34000000     3.34000000      .00000000
.99961029      3.36000000     3.36000000      .00000000
.99963757      3.38000000     3.38000000      .00000000
.99966307      3.40000000     3.40000000      .00000000
.99968689      3.42000000     3.42000000      .00000000
.99970914      3.44000000     3.44000000      .00000000
.99972991      3.46000000     3.46000000      .00000000
.99974929      3.48000000     3.48000000      .00000000
.99976737      3.50000000     3.50000000      .00000000
.99978423      3.52000000     3.52000000      .00000000
.99979994      3.54000000     3.54000000      .00000000
.99981457      3.56000000     3.56000000      .00000000
.99982820      3.58000000     3.58000000      .00000000
.99984089      3.60000000     3.60000000      .00000000
.99985270      3.62000000     3.62000000      .00000000
.99986368      3.64000000     3.64000000      .00000000
.99987389      3.66000000     3.66000000      .00000000
.99988338      3.68000000     3.68000000      .00000000
.99989220      3.70000000     3.70000000      .00000000
.99990039      3.72000000     3.72000000      .00000000
.99990799      3.74000000     3.74000000      .00000000
.99991504      3.76000000     3.76000000      .00000000
.99992159      3.78000000     3.78000000      .00000000
.99992765      3.80000000     3.80000000      .00000000
.99993327      3.82000000     3.82000000      .00000000
.99993848      3.84000000     3.84000000      .00000000
.99994331      3.86000000     3.86000000      .00000000
.99994777      3.88000000     3.88000000      .00000000
.99995190      3.90000000     3.90000000      .00000000
.99995573      3.92000000     3.92000000      .00000000
.99995926      3.94000000     3.94000000      .00000000
.99996252      3.96000000     3.96000000      .00000000
.99996554      3.98000000     3.98000000      .00000000
```

## BETA DISTRIBUTION

A random variable is said to have a beta distribution if its probability density is:

$$f(x) = \begin{cases} k.X^{a-1}\ (1 - X)^{b-1} & \text{for } 0 < X < 1 \\ 0 & \text{elsewhere} \end{cases}$$

where $k = \dfrac{\Gamma(a + b)}{\Gamma(a)\ \Gamma(b)}$

It is obvious from the above that the beta distribution is a two-parameter function and the distribution is related to the gamma distributions.

The beta distribution is fundamental in the solution of the I and J integrals used in total sediment in transport analysis. This distribution is also fundamental in the determination of the distribution of sediment deposited in a reservoir. This distribution is used extensively when the argument "X" (or its transform) is in the range zero to one.

The integral of f(x) is evaluated from "0" to "x" with the use of the following derivation:

Let the beta density function be defined as:

$$f(x) = \frac{X^{a-1} (1-X)^{b-1}}{\beta(a,\ b)} \quad \text{where } 0 \leq X \leq 1 \text{ and}$$

$$a > 0,\ b > 0$$

and the beta distribution function be defined as:

$$F(x) = \frac{1}{\beta(a,\ b)} \int_0^x t^{a-1} (1 - t^{b-1} dt$$

where

$$\beta(a,b) = \int_0^1 t^{a-1} (1 - t)^{b-1} dt = \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(a + b)} \qquad (1)$$

See 6.2.1, page 258, Handbook of Mathematical Functions, U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series 55, June 1964. On page 263, 6.6.1, of this same handbook the incomplete beta function is:

$$\beta x(a,\ b) = \int_0^x t^{a-1} (1 - t)^{b-1} dt$$

and

$$F(x) = I_x (a,\ b) = \beta x(a,\ b)/\beta(a,\ b) \qquad (2)$$

and

$$\beta x(a,\ b) = a^{-1} X^a F(a,\ 1-b;\ a+1;\ X)$$

where $F(a,\ 1-b;\ a+1;\ X)$ is the hypergeometric series.

The circle of convergence of the Gauss hypergeometric series is $|Z| < 1$. Within this circle F is defined as shown:

$$F(a,\ b;\ c;\ z) = \frac{\Gamma(c)}{\Gamma(a)\ \Gamma(b)} \sum_{n=o}^{\infty} \frac{\Gamma(a+n)\ \Gamma(b+n)\ Z^n}{\Gamma(c+n)\ n!}$$

$$= \frac{\Gamma(c)}{\Gamma(a)\ \Gamma(b)} \left[ \frac{\Gamma(a)\ \Gamma(b)\ Z^o}{\Gamma(c)\ 0!} + \frac{\Gamma(a+1)\ \Gamma(b+1)\ Z}{\Gamma(c+1)\ 1!} + \right.$$

$$\frac{\Gamma(a+2)\ \Gamma(b+2)\ Z^2}{\Gamma(c+2)\ 2!} + \frac{\Gamma(a+3)\ \Gamma(b+3)\ Z^3}{\Gamma(c+3)\ 3!}$$

$$\left. + \ \ldots \right]$$

Using recurrence formulas:

$\Gamma(z+1) = Z\Gamma(z)$ and

$\Gamma(z+n) = (n-1 + z)\ (n - 2 + z)\ \ldots\ (1 + Z)\ \Gamma(1 + z)$

then:

$$F(a,\ b;\ c;\ z) = \frac{\Gamma(c)}{\Gamma(a)\ \Gamma(b)} \left[ \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(c)} + \frac{a\Gamma(a)\ b\Gamma(b)\ Z}{c\Gamma(c)} + \right.$$

$$\frac{a(a+1)\ \Gamma(a)\ b\ (b+1)\ \Gamma(b)\ Z^2}{c(c+1)\ \Gamma(c)\ 2!} +$$

$$\left. \frac{a(a+1)(a+2)\ \Gamma(a)\ b\ (b+1)(b+2)\ \Gamma(b)\ Z^3}{c(c+1)(c+2)\ \Gamma(c)\ 3!} + \ \ldots \right]$$

and upon further reduction:

$$F(a,\ b;\ c;\ z) = \frac{\Gamma(c)}{\Gamma(a)\ \Gamma(b)} \left[ \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(c)} + \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(c)}\ \frac{abz}{c} + \right.$$

$$\frac{\Gamma(a)\ \Gamma(b)}{\Gamma(c)}\ \frac{a(a+1)\ b\ (b+1)\ Z^2}{c(c+1)\ 2!} +$$

$$\left. \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(c)}\ \frac{a(a+1)(a+2)\ b(b+1)(b+2)Z^3}{c(c+1)(c+2)\ 3!} + \ldots \right]$$

and:

$$F(a,\ b;\ c;\ z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \cdot \frac{\Gamma(a)\ \Gamma(b)}{\Gamma(c)} \left[ 1 + \frac{abz}{\Gamma(c)} + \frac{a(a+1)\ b(b+1)\ Z^2}{c(c+1)\ 2!} \right.$$

$$+ \frac{a(a+1)(a+2)b(b+1)(b+2)\ Z^3}{c(c+1)\ (c+2)\ 3!}$$

$$\left. + \ldots \right]$$

Let: $a = a$, $b = 1 - b$, $c = a + 1$, and $Z = X$

then:

$$F(a,\ 1\text{-}b;\ a+1;\ x) = 1 + \frac{a(1-b)X}{a+1} + \frac{a(a+1)(1-b)(1 - b + 1)\ X^2}{(a+1)(a + 1 + 1)\ 2!}$$

$$+ \frac{a(a+1)\ (a+2)\ (1-b)\ (1-b+1)\ (1-b+2)\ X^3}{(a+1)\ (a+1+1)\ (a+1+2)\ 3!} + \ldots$$

and:

$$F(a,\ 1\text{-}b;\ a+1;\ X) = 1 + \frac{a(1-b)\ X}{(a+1)} + \frac{a(1-b)(2-b)\ X^2}{(a+2)\ 2!} +$$

$$\frac{a(1-b)\ (2-b)\ (3-b)\ X^3}{(a+3)\ 3!} + \ldots$$

then:

$$\beta x(a,b) = a^{-1} X^{a} \left[ 1 + a \sum_{n=1}^{\infty} \frac{(n-b)^{(n)} X^{n}}{(n+a)\; n!} \right] \qquad (3)$$

where b = 1, 2, 3, 4, ...

Substituting equations (1) and (3) into equation (2)

$$I_x\,(a,\,b) = \frac{\Gamma(a+b)}{\Gamma(a)\;\Gamma(b)}\; a^{-1} X^{a} \left[ 1 + a \sum_{n=1}^{\infty} \frac{(n-b)^{(n)}\, X^{n}}{(n+a)\; n!} \right] \qquad (4)$$

Equation (4) was used to evaluate the incomplete beta function for all values of "a" and "b" over the interval X = 0, X = 1. The results of this evaluation were compared with those shown in Tables of the Incomplete Beta-Function, by Karl Pearson, F.R.S., Cambridge, Published for the Biometrika Trustees at the University Press, reprinted in 1956. The comparison was identical when "b" was an integer; however, for one-half units of "a" with one-half units of "b" the above solution of the integral deteriorated for $X > 0.50$. By taking advantage of the symmetry which is expressed as follows:

$$I_x(a,\,b) = 1 - I_{x-1}\,(b,\,a) \qquad (5)$$

a good comparison was obtained. In the data analysis model the integral was solved using the series of equation (4) and the symmetry of equation (5) where the series (4) was truncated when the last term was equal to $1 \cdot 10^{-7}$ or less. In the model "a" and "b" were allowed to assume any value greater than zero without any apparent discontinuity. Solving the inverse of the integral can be accomplished using Newton's method and will be described in a later portion of this narrative.

It is left to the experience of the user to use this technique with good judgment.

## Solving for mean and variance of the beta function given "a" and "b", where a > 0 and b > 0

The general term for the derivation of moments is as follows:

$$\mu_r = \frac{\Gamma(a+b) \quad \Gamma(a+r)}{\Gamma(a)\ \Gamma(a + b + r)}$$

where $r = 1, 2, 3, \ldots$ or the first, second, third ... moments.

When $r = 1$ the first moment or mean is:

$$\mu_1 = \frac{\Gamma(a+b)\ \Gamma(a+1)}{\Gamma(a)\ \Gamma(a + b + 1)}$$

Let: $a + b = c$

and the recurrence formula:

$$\Gamma(z + 1) = Z\ \Gamma(z)$$

then

$$\mu_1 = \frac{\Gamma(c)\ a\ \Gamma(a)}{\Gamma(a)\ c\ \Gamma(c)} = a/c = \frac{a}{a+b} = \text{mean.}$$

the second moment = $\mu_2$ and:

$$\mu_2 = \frac{\Gamma(a+b)\ \Gamma(a+2)}{\Gamma(a)\ \Gamma(a+b+2)}$$

Let: $a + b = c$

and using the recurrence formulas

$$\Gamma(z + 1) = Z\Gamma(z)$$

$$\Gamma(z + n) = (n - 1 + z)\ (n - 2 + z) \ldots (1 + z)\ \Gamma(z + 1)$$

then

$$\mu_2 = \frac{\Gamma(c)\ a\ (a+1)\ \Gamma(a)}{\Gamma(a)\ c\ (c+1)\ \Gamma(c)} = \frac{a(a+1)}{c(c+1)}$$

or

$$\mu_2 = \frac{a(a+1)}{(a+b)\ (a + b + 1)}$$

Variance = $\sigma^2 = E(x^2) - \mu_1^2$ and $E(x^2) = \mu_2$

Then:

$$\sigma^2 = \frac{a(a+1)}{(a+b)\ (a + b + 1)} - \frac{a^2}{(a+b)^2}$$

and:

$$\sigma^2 = \frac{a(a+b)\ (a+1) - a^2(a + b + 1)}{(a+b)^2\ (a + b + 1)}$$

which reduces to:

$$\sigma^2 = \frac{ab}{(a+b)^2\ (a + b + 1)} = \text{variance.}$$

Solving for "a" and "b" in terms of mean and variance

Solve first for "b" in terms of "a" and $\mu_1$

$$b = \frac{a(1 - \mu_1)}{\mu_1}$$

Solve for "a" by substitution of "b" in equation for variance

$$a = \mu_1 \left[ \frac{(\mu_1 - \mu_1^2)}{\sigma^2} - 1 \right] .$$

## Inverse of Integral

The inverse of the integral of the beta distribution function was first attempted using Newton's method. The computation of the inverse in this manner proved to be very sensitive at the end points and also lacked continuity for large values of the parameters "a" and "b". The mechanism used was as follows:

Let: $ZX = X_t^{a-1}\,(1 - X_t)^{b-1}$

$FN_t$ = value of integral for $X_t$

$FN_p$ = true value of integral

Then:

$$X_{t+1} = X_t - \frac{FN_t - FN_p}{ZX} \quad \text{(Newton's method)}$$

The final attempt for the solution of inverse of the aforementioned integral made use of the following recursion formulas:

$$I_x(a,b) = I_x(a,b-1) + \frac{x^a(1-x)^{b-1}}{(a+b-1)\beta(a,b)}$$

and: $I_x(a,a) = 1/2\,[1 + I_{x'}(1/2,a)]$

$= 1 - 1/2\; I_{1-x'}\;(a,1/2)$

where: $x' = 4(x - 1/2)^2$ or $x = 1/2(1 + \sqrt{x'}$

The technique used to estimate the value of the inverse then became:

Let: $FN_p$ = true value of integral

$FN' = X_n - X_{n-1}$

$FN'' = FN_{x_n} - FN_{x_{n-1}}$ (values of integral for $X_n$ and $X_{n-1}$)

Then: $X_{n+1} = X_n - \left[ FN'(IN_{x_n} - FN_p)/FN'' \right]$

The subsequent narrative includes the computer generated tables for a range of distribution parameters "a" and "b". The table is self-explanatory and the values shown therein were compared with those included in K. Pearson's Tables of the Incomplete Beta Function, Cambridge (1956). This comparison indicated the computer generated table was within all significant digits of those of Pearson's Tables.

## SUBROUTINE BETAX

This subroutine is the electronic computer application for the solution of the beta distribution function and its inverse.

The entry for the solution of the inverse is labeled BETAX1.

The arguments for entry are labeled as follows:

A = value of a

B = value of b

XR = value of x

BETA = value of distribution function and true value for the inverse computation

XC = value of the inverse

The label CONGAM is the title of the computer function for the solution of the complete gamma function. This function (subroutine type) will be discussed in a subsequent portion of this narrative which concerns the subroutines required for the electronic computer application for the solution of the incomplete gamma function.

All other labels included in the BETAX subroutine are self-explanatory.

```
PROGRAM DRIVER      74/74    OPT=1                         FTN 4.2+P390       75/03/25. 09.12.28.       PAGE     1

            PROGRAM DRIVER(INPUT,OUTPUT,FIN,FOU,TAPE5=FIN,TAPE6=FOU)
            COMMON/UNITS/IW,IR
            COMMON/VARBLS/FX,XAB,FOFX
            IW=6 $ IR=5
            A=0.0 $ B=0.5 $ LINE=0
         10 A = A+ 5.E-1
         12 IF(A.GT.9.0) GO TO 100
            X=0.0
            IF(LINE.NE.0) GO TO 17
            WRITE(IW,15)
         15 FORMAT(1H1,///23X*INCOMPLETE BETA FUNCTION AND INVERSE*)
         17 WRITE(IW,19)A,B
         19 FORMAT(/23X*A = *F5.1* AND B = *F5.1/25X*X*5X*BETA(X)*
          1  5X*INVERSE XC*2X*ERROR TERM*)
            DO 25 K=1,25
            X = X + 4.E-2 $ IF(K.EQ.25) X=1.0
            CALL BETAX(A,B,X,BETA,XC)
            BETA1=BETA
            CALL BETAX1(A,B,X,BETA,XC)
            DIFF=X-XC
            IF(XC.EQ.99.) GO TO 21
            WRITE(IW,20)X,BETA1,XC,DIFF
         20 FORMAT(20X,F6.2,3(2X,F10.7))
            GO TO 25
         21 WRITE(IW,22)X,BETA1
         22 FORMAT(20X,F6.2,2X,F10.7,2(1X*NOT DEFINED*))
         25 CONTINUE
            LINE=LINE+1 $ IF(LINE.EQ.2)LINE=0
            GO TO 10
        100 IF ( B . GT. 5. ) GO TO 200
            B = B + 0.5
            A = B
            GO TO 12
        200 CALL EXIT
            END

PROGRAM DRIVER      74/74    OPT=1                         FTN 4.2+P390       75/03/25. 09.12.28.       PAGE     2

SYMBOLIC REFERENCE MAP (R=1)
```

```
          PROGRAM DRIVER     74/74   OPT=1                               FTN 4.2+P380       75/03/25. 09.12.28.      PAGE     2

       SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
10214  DRIVER

VARIABLES     SN  TYPE            RELOCATION
10366  A          REAL                          10367  B         REAL
10373  BETA       REAL                          10375  BETA1     REAL
10376  DIFF       REAL                              2  FOFX      REAL                VARBLS
    0  FX         REAL                VARBLS        1  IR        INTEGER             UNITS
    0  IM         INTEGER             UNITS     10372  K         INTEGER
10377  LINE       INTEGER                       10371  X         REAL
    1  XAD        REAL                VARBLS    10374  XC        REAL

FILE NAMES        MODE
 4102  FIN                      6143  FOU                        0  INPUT                   2041  OUTPUT
 4102  TAPE5                    6143  TAPE6

EXTERNALS          TYPE   ARGS
       BETAX                5                           BETAX1               5
       EXIT                 0

STATEMENT LABELS
10222  10                               10224  12                               10306  15      FMT
10232  17                               10322  19     FMT                       10343  20      FMT
10255  21                               10353  22     FMT                       10257  25
10266  100                              10273  200

COMMON BLOCKS   LENGTH
       UNITS         2
       VARBLS        3

STATISTICS
  PROGRAM LENGTH                 173B     123
  BUFFER LENGTH                10204B    4228
  CM LABELED COMMON LENGTH         5B       5
```

```
SUBROUTINE BETAX       74/74    OPT=1                          FTN 4.2+P380       75/03/25. 09.12.29.       PAGE     1

            SUBROUTINE BETAX(A,B,XR,BETA,XC)
            COMMON/UNITS/IW,IR
            COMMON/VARBLS/FX,XAB,FOFX
            X=XR
            ASSIGN 2000 TO ISX
       10   IOW=0 $ XA=X $ IF(A.NE.B) GO TO 12
            IOW=1 $ BZ=0.5 $ TZ=BZ $ MAX=0
            XA=XA-0.5 $ XA=4.0*XA*XA $ XA=1.0-XA
            GO TO 15
       12   NP=B $ BP=NP $ BZ=B-BP $ TZ=BZ
            MAX=NP  $ IF(BZ.GT.1.0E-10) GO TO 15
            MAX=MAX-1 $ BZ=1.0 $ TZ=1.0
       15   IF(XA.GT.0.5) GO TO 30
            AB=A $ BC=BZ $ ISW=0
            GO TO 50
       30   AB=BZ $ BC=A $ XA=1.0-XA $ ISW=1
       50   GX=AB $ FACTN=1.0 $ FOFX=1.0 $ BETAM=1.0
            DO 60 K=1,60
            BK=K $ FACTN=FACTN*BK $ BD=BK-BC
            GX=GX*XA*BD $ FX=GX/((AB+BK)*FACTN)
            FOFX=FOFX+FX $ IF(ABS(FX).LT.1.0E-10) GO TO 90
       60   CONTINUE
            PRINT 70
       70   FORMAT(1X*NO CONVERGENCE FOUND FOR FX - ABORTED*)
            CALL EXIT
       90   XM=ABS(XA) $ BETAM=XM**AB/AB $ ARGA=AB+BC
            XAB=CONGAM(ARGA)/(CONGAM(AB)*CONGAM(BC))
            BETAM=XAB*BETAM*FOFX $ IF(ISW.EQ.1)BETAM=1.0-BETAM
            IF(BETAM.LE.1.0E-14) BETAM=0.0
            IF(MAX.NE.0) GO TO 100
            GO TO 150
      100   XAP=X**A $ CONA=CONGAM(A)
            DO 110 K=1,MAX
            CK=K $ CKK=CK+TZ $ ARGA=A+CKK $ CONB=CONGAM(CKK)
            XAB=CONGAM(ARGA)/(CONA*CONB) $ XAQ=1.0-X $ CKK=CKK-1.0
            XAQ=XAQ**CKK $ XAC=A+CKK $ BETAM=BETAM+(XAP*XAQ*XAB/XAC)
       95   FORMAT(1X*XAQ = *F8.6,2X*XAC =*F8.6,2X*XAP = *F8.6
          1  /1X*XAB = *F15.6)
      110   CONTINUE
      120   FORMAT(1X*MAX = *I4,5X*X = *F15.7,5X*BZ = *F15.7
          1  /1X*A = *F15.7,2X*B = *F15.7,2X*BETA = *F15.7)
      150   BETA=BETAM $ IF(IOW.NE.1) GO TO 160
            BETA=1.0-(0.5*BETAM) $ IF(X.LT.0.5)BETA=1.0-BETA
      160   GO TO ISX(2000,250,300,310)
            ENTRY BETAX1
            X=1.0 $ IF(BETA.EQ.1.0) GO TO 450
            X=1.0-1.0E-5 $ NC=0 $ TEST=BETA $ SAVE=X
            IF(BETA.GT.1.0E-7) GO TO 200
            X=99. $ GO TO 450
      200   ASSIGN 250 TO ISX $ GO TO 10
      250   IF(NC.GT.0) GO TO 300
            IF(BETA.LT.TEST) GO TO 280
            IF(BETA.EQ.TEST) GO TO 450
            SAVE=X $ X=X-1.0E-1 $ IF(X.LE.0.0) GO TO 280
            SAVE=X $ GO TO 200
      280   X=X1=SAVE $ ASSIGN 300 TO ISX $ GO TO 10
      300   FX1=BETA $ X=X2=X1-1.0E-7



SUBROUTINE BETAX       74/74    OPT=1                          FTN 4.2+P380       75/03/25. 09.12.29.       PAGE     2

      305   ASSIGN 310 TO ISX $ GO TO 10
      310   FX2=BETA $ DX=X2-X1 $ DXDP=FX2-FX1
```

```
  305  ASSIGN 310 TO ISX $ GO TO 10
  310  FX2=BETA $ DX=X2-X1 $ DXDP=FX2-FX1
       IF(DXDP.EQ.0.0) GO TO 450
       X3=X2-(DX*(FX2-TEST)/DXDP) $ ERROR = X3-X2
       IF(ABS(ERROR).LT.1.0E-7) GO TO 450
       X1=X2 $ X=X2=X3 $ NC=NC+1 $ FX1=FX2
       IF(X2.GT.X1*3.0)X=X2=3.0*X1
       IF(NC.LE.100) GO TO 305
  450  XC=X $ GO TO 2000
 2000  RETURN
       END
```

```
     SUBROUTINE BETAX      74/74   OPT=1                     FTN 4.2+P390       75/03/25. 09.12.29.      PAGE     3

     SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
    3  BETAX          206  BETAX1

VARIABLES      SN  TYPE            RELOCATION
    0  A           REAL                 F.P.        402  AB          REAL
  414  ARGA        REAL                               0  B           REAL                F.P.
  403  BC          REAL                             412  BD          REAL
    0  BETA        REAL                 F.P.        407  BETAM       REAL
  411  BX          REAL                             401  BP          REAL
  375, BZ          REAL                             417  CK          REAL
  420  CKK         REAL                             416  CONA        REAL
  421  CONB        REAL                             433  DX          REAL
  434  DXDP        REAL                             436  ERROR       REAL
  436  FACTN       REAL                               2  FOFX        REAL                VARBLS
    0  FX          REAL               VARBLS        430  FX1         REAL
  432  FX2         REAL                             405  GX          REAL
  373  ISW         INTEGER                            1  IR          INTEGER             UNITS
  404  ISW         INTEGER                          372  ISX         INTEGER
    0  IW          INTEGER            UNITS         410  K           INTEGER
  377  MAX         INTEGER                          424  NC          INTEGER
  400  NP          INTEGER                          426  SAVE        REAL
  425  TEST        REAL                             376  TZ          REAL
  371  X           REAL                             374  XA          REAL
    1  XAB         REAL               VARBLS        423  XAC         REAL
  415  XAP         REAL                             422  XAQ         REAL
    0  XC          REAL                 F.P.        413  XM          REAL
    0  XR          REAL                 F.P.        427  X1          REAL
  431  X2          REAL                             435  X3          REAL

FILE NAMES        MODE
      OUTPUT     FMT

EXTERNALS            TYPE   ARGS
      CONGAM     REAL      1                       EXIT                0

INLINE FUNCTIONS    TYPE   ARGS
      ABS        REAL      1  INTRIN

STATEMENT LABELS
   11  10                          25  12                                    37  15
   45  30                          53  50                                     0  60
  324  70      FMT                103  90                                   332  95      FMT  NO REFS
  134  100                          0  110                                  342  120     FMT  NO REFS
  173  150                        203  160                                  227  200
  231  250                        243  280                                  247  300
  253  305                        255  310                                  302  450
  304  2000

COMMON BLOCKS    LENGTH
      UNITS           2
      VARBLS          1

STATISTICS
  PROGRAM LENGTH                4378      287
  CM LABELED COMMON LENGTH        5B        5




       FUNCTION CONGAM     74/74   OPT=1                     FTN 4.2+P390       75/03/25. 09.12.31.      PAGE     1

                  FUNCTION CONGAM  (ALF )
                  COMMON/UNITS/IW,IR
                  DIMENSION   BC (8)
```

```
FUNCTION CONGAM     74/74    OPT=1                          FTN 4.2+P390       75/03/25. 09.12.31.       PAGE    1

              FUNCTION CONGAM  (ALF )
              COMMON/UNITS/IN,IR
              DIMENSION   BC (8)
              DATA((BC (K) , K = 1,8 )  =   -0.577191652 , 0.988205891 ,
             1   -0.897056937  , 0.918206857 ,  - 0.756704078  , 0.482199394,
             2     -0.193527818  ,   0.035868343    )
              PI = 3.14159265358979
              KSW = 1
              CONGAM = 0.0
              IF ( ALF . GT. 1. E-10  )  GO TO 1
              IF ( ALF . LT. 1. E-10  )  GO TO 5
              CONGAM = 1.0
              GO TO 100
            5 NCP = ALF
              SAVE = ALF
              ALF1 = ABS (NCP)
              COMP = 0.0
              COMP = ALF + ALF1
              IF ( ABS (COMP) .GT. 1.E-10 )    GO TO 15
              PRINT 10
           10 FORMAT ( 8X,  28HALPHA IS A NEGATIVE INTEGER   )
              GO TO 100
           15 KSW = 2
              ALF = 1.0 - ALF
              IF ( ALF . GT. 1.E-10  )  GO TO 1
              PRINT 19
           19 FORMAT ( 8X, 30HBAD EQUATION IN TERMS OF ALPHA   )
              CALL EXIT
           13 ALF = SAVE
              CONGAM = PI / ( CONGAM * SIN ( PI * ALF) )
              GO TO 100
            1 MARK = 1
              FR = 1.0
              N = ALF
              COMP = 0.0
              COMP=ALF+1.0E-10
              N = COMP
         CK D3(" C1    8 FORMAT ( 8X, I4 , 8X, E19.12 )
              KN = N - 1
              IF (KN.GE.1)  GO TO 20
              IF ( ALF .LT. 1.0 )  GO TO 9
              X = ALF - 1.0
              GO TO 40
            9 MARK = 2
              X = ALF
              GO TO 40
           20 DO 30  K = 1, KN
              FK = K
              FA = 0.0
              FA = ALF - FK
              FR = FR * FA
           30 CONTINUE
              X = FA - 1.0
              TEST = ABS (X)
              IF (TEST.GE.1.E-8)  GO TO 40
              FOFX = 1.0
              GO TO 60
```

```
FUNCTION CONGAM     74/74   OPT=1                    FTN 4.2+P380      75/03/25. 09.12.31.      PAGE     2

                40 JB = 8
                   FOFX = 0.0
                50 FOFX = FOFX + BC(JB)
                   FOFX = FOFX * X
                   JB = JB - 1
                   IF (JB.GT.0)  GO TO 50
                   FOFX = FOFX + 1.0
                60 CONGAM = FOFX * FN
                   IF (MARK .EQ. 2 ) CONGAM = CONGAM / X
                   IF ( KSW .EQ. 2 )  GO TO 19
               100 RETURN
                   END

       FUNCTION CONGAM     74/74   OPT=1                    FTN 4.2+P380      75/03/25. 09.12.31.      PAGE     3

CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM
                           BASIC EXTERNAL OR INTRINSIC FUNCTION CALLED WITH WRONG TYPE ARGUMENT.
```

```
      FUNCTION CONGAM     74/74   OPT=1                          FTN 4.2+P380      75/03/25. 09.12.31.      PAGE     3
CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM
     16    I               BASIC EXTERNAL OR INTRINSIC FUNCTION CALLED WITH WRONG TYPE ARGUMENT.


      FUNCTION CONGAM     74/74   OPT=1                          FTN 4.2+P380      75/03/25. 09.12.31.      PAGE     4

         SYMBOLIC REFERENCE MAP (R=1)
```

```
        FUNCTION CONGAM     74/74    OPT=1                          FTN 4.2+P380       75/03/25. 09.12.31.      PAGE    4

      SYMBOLIC REFERENCE MAP (R=1)

  ENTRY POINTS
      4  CONGAM

  VARIABLES     SN  TYPE            RELOCATION
      0  ALF         REAL                F.P.          160  ALF1      REAL
    175  BC          REAL        ARRAY                 161  COMP      REAL
    153  CONGAM      REAL                              171  FA        REAL
    163  FB          REAL                              170  FK        REAL
    173  FOFX        REAL                                1  IR        INTEGER          UNITS
      0  IW          INTEGER            UNITS          174  JB        INTEGER
    167  K           INTEGER                           165  KN        INTEGER
    155  KSW         INTEGER                           162  MARK      INTEGER
    164  N           INTEGER                           156  NCP       INTEGER
    154  PI          REAL                              157  SAVE      REAL
    172  TEST        REAL                              166  X         REAL

  FILE NAMES        MODE
        OUTPUT      FMT

  EXTERNALS           TYPE   ARGS
        EXIT                   0                       SIN       REAL       1 LIBRARY

  INLINE FUNCTIONS   TYPE   ARGS
        ABS          REAL      1  INTRIN

  STATEMENT LABELS
     45  1                             16  5                                     62  9
    132  10      FMT                   30  15                                   142  18      FMT
     37  19                            65  20                                     0  30
    105  40                           107  50                                   116  60
    126  100

  COMMON BLOCKS   LENGTH
        UNITS          2

  STATISTICS
    PROGRAM LENGTH                2058      133
    CM LABELED COMMON LENGTH        28        2
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =      .5 AND B =      .5
  X      BETA(X)        INVERSE XC   ERROR TERM
 .04     .1281884      .0400000     -.0000000
 .08     .1825549      .0800000      .0000000
 .12     .2251989      .1200000      .0000000
 .16     .2619798      .1600000      .0000000
 .20     .2951673      .2000000      .0000000
 .24     .3259320      .2400000      .0000000
 .28     .3549785      .2800000      .0000000
 .32     .3827767      .3200000      .0000000
 .36     .4096655      .3600000      .0000000
 .40     .4359058      .4000000      .0000000
 .44     .4617105      .4400000      .0000000
 .48     .4872642      .4800000      .0000000
 .52     .5127358      .5200000     -.0000000
 .56     .5382895      .5600000      .0000000
 .60     .5640942      .6000000     -.0000000
 .64     .5903345      .6400000      .0000000
 .68     .6172233      .6800000      .0000000
 .72     .6450215      .7200000      .0000000
 .76     .6740680      .7600000     -.0000000
 .80     .7048327      .8000000     -.0000000
 .84     .7330202      .8400000     -.0000000
 .88     .7748011      .8800000     -.0000000
 .92     .8174451      .9200000     -.0000000
 .96     .8718116      .9600000      .0000000
1.00    1.0000000     1.0000000     0.0000000

A =     1.0 AND B =      .5
  X      BETA(X)        INVERSE XC   ERROR TERM
 .04     .0202041      .0400000     -.0000000
 .08     .0408337      .0800000     -.0000000
 .12     .0619168      .1200000      .0000000
 .16     .0834849      .1600000      .0000000
 .20     .1055728      .2000000     -.0000000
 .24     .1282202      .2400000      .0000000
 .28     .1514719      .2800000      .0000000
 .32     .1753789      .3200000      .0000000
 .36     .2000000      .3600000      .0000000
 .40     .2254033      .4000000     -.0000000
 .44     .2516685      .4400000      .0000000
 .48     .2788897      .4800000     -.0000000
 .52     .3071797      .5200000      .0000000
 .56     .3366750      .5599999      .0000001
 .60     .3675445      .6000000     -.0000000
 .64     .4000000      .6400000      .0000000
 .68     .4343146      .6800000     -.0000000
 .72     .4708497      .7200000      .0000000
 .76     .5101021      .7600000     -.0000000
 .80     .5527864      .8000000     -.0000000
 .84     .6000000      .8400000     -.0000000
 .88     .6535898      .8800000      .0000000
 .92     .7171573      .9200000     -.0000000
 .96     .8000000      .9600000      .0000000
1.00    1.0000000     1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     1.5 AND B =      .5
  X        BETA(X)          INVERSE XC   ERROR TERM
 .04      .0034369        .0400000     -.0000000
 .08      .0098443        .0800000     -.0000000
 .12      .0183220        .1200000     -.0000000
 .16      .0285911        .1600000     -.0000000
 .20      .0405193        .2000000     -.0000000
 .24      .0540424        .2400000     -.0000000
 .28      .0691369        .2800000     -.0000000
 .32      .0858087        .3200000      .0000000
 .36      .1040880        .3600000     -.0000000
 .40      .1240270        .4000000     -.0000000
 .44      .1457008        .4400000     -.0000000
 .48      .1692090        .4800000     -.0000000
 .52      .1946808        .5200000      .0000000
 .56      .2222798        .5600000     -.0000000
 .60      .2522156        .6000000     -.0000000
 .64      .2847571        .6400000     -.0000000
 .68      .3202555        .6800000     -.0000000
 .72      .3591801        .7200000      .0000000
 .76      .4021786        .7600000     -.0000000
 .80      .4501849        .8000000     -.0000000
 .84      .5046317        .8400000     -.0000000
 .88      .5679242        .8800000      .0000000
 .92      .6447346        .9200000     -.0000000
 .96      .7470601        .9599999      .0000001
1.00     1.0000000       1.0000000     0.0000000

A =     2.0 AND B =      .5
  X        BETA(X)          INVERSE XC   ERROR TERM
 .04      .0006082        .0400000     -.0000000
 .08      .0024670        .0800000     -.0000000
 .12      .0056319        .1200000     -.0000000
 .16      .0101636        .1600000      .0000000
 .20      .0161301        .2000000     -.0000000
 .24      .0236066        .2400000     -.0000000
 .28      .0326779        .2800000      .0000000
 .32      .0434395        .3199999      .0000001
 .36      .0560000        .3600000     -.0000000
 .40      .0704840        .4000000     -.0000000
 .44      .0870356        .4400000     -.0000000
 .48      .1058233        .4800001     -.0000001
 .52      .1270464        .5200000      .0000000
 .56      .1509441        .5600000     -.0000000
 .60      .1778078        .6000000     -.0000000
 .64      .2080000        .6400000     -.0000000
 .68      .2419815        .6800000     -.0000000
 .72      .2803556        .7200000      .0000000
 .76      .3239408        .7600000     -.0000000
 .80      .3739010        .8000000     -.0000000
 .84      .4320000        .8400000     -.0000000
 .88      .5011694        .8800000      .0000000
 .92      .5870496        .9200000     -.0000000
 .96      .7040000        .9600000      .0000000
1.00     1.0000000       1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    2.5 AND B =      .5
  X       BETA(X)       INVERSE XC   ERROR TERM
 .04      .0001102      .0400000    -.0000000
 .08      .0006330      .0800001    -.0000001
 .12      .0017718      .1200000    -.0000000
 .16      .0036963      .1600000     .0000000
 .20      .0065663      .2000000    -.0000000
 .24      .0105400      .2400001    -.0000001
 .28      .0157798      .2800000     .0000000
 .32      .0224556      .3200000    -.0000000
 .36      .0307494      .3600000     .0000000
 .40      .0408594      .4000000    -.0000000
 .44      .0530046      .4400000    -.0000000
 .48      .0674314      .4800000     .0000000
 .52      .0844217      .5199999     .0000001
 .56      .1043029      .5600000    -.0000000
 .60      .1274641      .6000000    -.0000000
 .64      .1543774      .6400000    -.0000000
 .68      .1856300      .6800000     .0000000
 .72      .2219762      .7200000     .0000000
 .76      .2644212      .7600000    -.0000000
 .80      .3143727      .8000000    -.0000000
 .84      .3739340      .8400000    -.0000000
 .88      .4465564      .8800000     .0000000
 .92      .5388054      .9200000    -.0000000
 .96      .6672192      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000

A =    3.0 AND B =      .5
  X       BETA(X)       INVERSE XC   ERROR TERM
 .04      .0000203      .0400000    -.0000000
 .08      .0001650      .0800000    -.0000000
 .12      .0005662      .1200000     .0000000
 .16      .0013651      .1599999     .0000001
 .20      .0027137      .2000000    -.0000000
 .24      .0047762      .2400000     .0000000
 .28      .0077312      .2800000     .0000000
 .32      .0117740      .3200000    -.0000000
 .36      .0171200      .3600000     .0000000
 .40      .0240082      .4000000    -.0000000
 .44      .0327067      .4400000    -.0000000
 .48      .0435194      .4800000     .0000000
 .52      .0567944      .5200000    -.0000000
 .56      .0729370      .5600000     .0000000
 .60      .0924263      .6000000    -.0000000
 .64      .1158400      .6400000    -.0000000
 .68      .1438917      .6800000     .0000000
 .72      .1774888      .7199999     .0000001
 .76      .2178289      .7600001    -.0000001
 .80      .2665697      .8000000    -.0000000
 .84      .3261600      .8400000    -.0000000
 .88      .4005719      .8800000     .0000000
 .92      .4972754      .9200000    -.0000000
 .96      .6348800      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000
```

# INCOMPLETE BETA FUNCTION AND INVERSE

```
A =    3.5 AND B =      .5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000038       .0400000    -.0000000
 .08     .0000435       .0800000    -.0000000
 .12     .0001830       .1200000     .0000000
 .16     .0005098       .1600000    -.0000000
 .20     .0011338       .2000000    -.0000000
 .24     .0021876       .2400000     .0000000
 .28     .0038278       .2800000     .0000000
 .32     .0062372       .3200000    -.0000000
 .36     .0096279       .3600000     .0000000
 .40     .0142458       .4000000    -.0000000
 .44     .0203755       .4400000    -.0000000
 .48     .0283488       .4800000     .0000000
 .52     .0385539       .5200000    -.0000000
 .56     .0514492       .5600000     .0000000
 .60     .0675834       .6000000    -.0000000
 .64     .0876230       .6400000    -.0000000
 .68     .1123938       .6800000     .0000000
 .72     .1429467       .7200000    -.0000000
 .76     .1806648       .7600000     .0000000
 .80     .2274529       .8000000    -.0000000
 .84     .2861052       .8400000    -.0000000
 .88     .3611135       .8800000     .0000000
 .92     .4608415       .9200000    -.0000000
 .96     .6059013       .9600000     .0000000
1.00    1.0000000      1.0000000     0.0000000

A =    4.0 AND B =      .5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000007       .0400000    -.0000000
 .08     .0000116       .0800000    -.0000000
 .12     .0000596       .1200000     .0000000
 .16     .0001920       .1600000     .0000000
 .20     .0004776       .2000000    -.0000000
 .24     .0010101       .2400000     .0000000
 .28     .0019103       .2800000    -.0000000
 .32     .0033299       .3200000    -.0000000
 .36     .0054560       .3599999     .0000001
 .40     .0085163       .4000000    -.0000000
 .44     .0127861       .4400000     .0000000
 .48     .0185978       .4800000     .0000000
 .52     .0263519       .5200000    -.0000000
 .56     .0365338       .5600000     .0000000
 .60     .0497356       .6000000    -.0000000
 .64     .0666880       .6400000    -.0000000
 .68     .0883074       .6800000     .0000000
 .72     .1157687       .7200000    -.0000000
 .76     .1506247       .7600000     .0000000
 .80     .1950155       .8000000    -.0000000
 .84     .2520720       .8400000    -.0000000
 .88     .3268003       .8799999     .0000001
 .92     .4284484       .9200000    -.0000000
 .96     .5795840       .9600000     .0000000
1.00    1.0000000      1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    4.5 AND B =     .5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000001     .0400000    -.0000000
 .08     .0000031     .0800001    -.0000001
 .12     .0000196     .1199999     .0000001
 .16     .0000728     .1599999     .0000001
 .20     .0002025     .2000000    -.0000000
 .24     .0004694     .2400000     .0000000
 .28     .0009593     .2800000     .0000000
 .32     .0017887     .3200000    -.0000000
 .36     .0031104     .3600000     .0000000
 .40     .0051211     .4000000    -.0000000
 .44     .0080697     .4400000     .0000000
 .48     .0122691     .4800000     .0000000
 .52     .0181100     .5200000    -.0000000
 .56     .0260794     .5600000     .0000000
 .60     .0367877     .6000000    -.0000000
 .64     .0510034     .6400001    -.0000001
 .68     .0697075     .6800000     .0000000
 .72     .0941742     .7200000    -.0000000
 .76     .1261034     .7600000     .0000000
 .80     .1678508     .8000000    -.0000000
 .84     .2228685     .8400001    -.0000001
 .88     .2966651     .8800000     .0000000
 .92     .3993615     .9200000    -.0000000
 .96     .5554455     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000

A =    5.0 AND B =     .5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000008     .0800000    -.0000000
 .12     .0000065     .1200000     .0000000
 .16     .0000277     .1600000     .0000000
 .20     .0000863     .2000000    -.0000000
 .24     .0002192     .2400000     .0000000
 .28     .0004842     .2800000     .0000000
 .32     .0009656     .3200000    -.0000000
 .36     .0017818     .3600000     .0000000
 .40     .0030941     .4000000    -.0000000
 .44     .0051167     .4400000     .0000000
 .48     .0081307     .4800000     .0000000
 .52     .0125005     .5200000    -.0000000
 .56     .0186962     .5600000     .0000000
 .60     .0273229     .6000000    -.0000000
 .64     .0391629     .6400000     .0000000
 .68     .0552348     .6800000     .0000000
 .72     .0768851     .7200000    -.0000000
 .76     .1059339     .7600000     .0000000
 .80     .1449276     .8000000    -.0000000
 .84     .1976173     .8400000     .0000000
 .88     .2699963     .8800000     .0000000
 .92     .3730427     .9200000    -.0000000
 .96     .5331354     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    5.5 AND B =      .5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000002      .0800000    -.0000000
 .12     .0000021      .1200000     .0000000
 .16     .0000106      .1600000    -.0000000
 .20     .0000369      .2000000    -.0000000
 .24     .0001028      .2400000     .0000000
 .28     .0002454      .2800000     .0000000
 .32     .0005234      .3200001    -.0000001
 .36     .0010248      .3600000     .0000000
 .40     .0018767      .4000000    -.0000000
 .44     .0032568      .4400000     .0000000
 .48     .0054084      .4799999     .0000001
 .52     .0086603      .5200000    -.0000000
 .56     .0134509      .5600000     .0000000
 .60     .0203632      .6000000    -.0000000
 .64     .0301709      .6400000     .0000000
 .68     .0439060      .6800000     .0000000
 .72     .0629598      .7200000    -.0000000
 .76     .0892442      .7600000     .0000000
 .80     .1254670      .8000000    -.0000000
 .84     .1756517      .8400000     .0000000
 .88     .2462522      .8800000     .0000000
 .92     .3490844      .9200000    -.0000000
 .96     .5123899      .9600000     .0000000
1.00    1.0000000     1.0000000    0.0000000

A =    6.0 AND B =      .5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000001 NOT DEFINED NOT DEFINED
 .12     .0000007      .1200000     .0000000
 .16     .0000041      .1600000    -.0000000
 .20     .0000159      .2000000    -.0000000
 .24     .0000484      .2399999     .0000001
 .28     .0001248      .2800000    -.0000000
 .32     .0002846      .3200000     .0000000
 .36     .0005914      .3600000    -.0000000
 .40     .0011421      .4000000    -.0000000
 .44     .0020796      .4400000     .0000000
 .48     .0036090      .4800000    -.0000000
 .52     .0060181      .5200000    -.0000000
 .56     .0097060      .5600000     .0000000
 .60     .0152201      .6000000    -.0000000
 .64     .0233084      .6400000     .0000000
 .68     .0349944      .6800000     .0000000
 .72     .0516885      .7200000    -.0000000
 .76     .0753654      .7600000     .0000000
 .80     .1088643      .8000000    -.0000000
 .84     .1564496      .8400000     .0000000
 .88     .2250075      .8800000     .0000000
 .92     .3271668      .9200000    -.0000000
 .96     .4930037      .9599999     .0000001
1.00    1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.5 AND B =     .5
  X       BETA(X)       INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000002     .1200000    -.0000000
 .16     .0000016     .1600000     .0000000
 .20     .0000068     .2000000    -.0000000
 .24     .0000228     .2400000    -.0000000
 .28     .0000636     .2800000     .0000000
 .32     .0001553     .3200000     .0000000
 .36     .0003423     .3600000    -.0000000
 .40     .0006970     .4000000    -.0000000
 .44     .0013316     .4400000     .0000000
 .48     .0024147     .4800000    -.0000000
 .52     .0041932     .5200000    -.0000000
 .56     .0070219     .5600000     .0000000
 .60     .0114045     .6000000    -.0000000
 .64     .0180502     .6400000     .0000000
 .68     .0279560     .6800000     .0000000
 .72     .0425286     .7200000    -.0000000
 .76     .0637778     .7600000     .0000000
 .80     .0946425     .8000000    -.0000000
 .84     .1395953     .8400000     .0000000
 .88     .2059218     .8800000     .0000000
 .92     .3070345     .9200000    -.0000000
 .96     .4748140     .9600000    -.0000000
1.00    1.0000000    1.0000000    0.0000000

A =    7.0 AND B =     .5
  X       BETA(X)       INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000001 NOT DEFINED NOT DEFINED
 .16     .0000006     .1600000     .0000000
 .20     .0000030     .2000000    -.0000000
 .24     .0000109     .2400000    -.0000000
 .28     .0000325     .2800000     .0000000
 .32     .0000849     .3200000     .0000000
 .36     .0001986     .3600000    -.0000000
 .40     .0004264     .4000000    -.0000000
 .44     .0008547     .4399999     .0000001
 .48     .0016194     .4800001    -.0000001
 .52     .0029281     .5200000    -.0000000
 .56     .0050911     .5600000     .0000000
 .60     .0085635     .6000000    -.0000000
 .64     .0140071     .6400000     .0000000
 .68     .0223778     .6800000     .0000000
 .72     .0350587     .7200000    -.0000000
 .76     .0540694     .7600000     .0000000
 .80     .0824179     .8000000    -.0000000
 .84     .1247504     .8400000     .0000000
 .88     .1887165     .8800000     .0000000
 .92     .2884781     .9200000    -.0000000
 .96     .4576879     .9600000    -.0000000
1.00    1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    7.5 AND B =      .5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000000 NOT DEFINED NOT DEFINED
 .16     .0000002     .1600000   -.0000000
 .20     .0000013     .2000000   -.0000000
 .24     .0000051     .2400001   -.0000001
 .28     .0000167     .2800000    .0000000
 .32     .0000465     .3200000    .0000000
 .36     .0001155     .3600000    .0000000
 .40     .0002614     .4000000   -.0000000
 .44     .0005497     .4400000    .0000000
 .48     .0010882     .4800000    .0000000
 .52     .0020490     .5200000   -.0000000
 .56     .0036986     .5600000    .0000000
 .60     .0064427     .6000000   -.0000000
 .64     .0108897     .6400000    .0000000
 .68     .0179443     .6800000   -.0000000
 .72     .0289497     .7200000   -.0000000
 .76     .0459122     .7600000    .0000000
 .80     .0718797     .8000000   -.0000000
 .84     .1116377     .8400000    .0000000
 .88     .1731612     .8800000   -.0000000
 .92     .2713245     .9200000   -.0000000
 .96     .4415161     .9600000   -.0000000
1.00    1.0000000    1.0000000   0.0000000

A =    8.0 AND B =      .5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000000 NOT DEFINED NOT DEFINED
 .16     .0000001 NOT DEFINED NOT DEFINED
 .20     .0000006     .2000000   -.0000000
 .24     .0000024     .2400000    .0000000
 .28     .0000086     .2800000    .0000000
 .32     .0000255     .3200000    .0000000
 .36     .0000672     .3600000    .0000000
 .40     .0001605     .4000000   -.0000000
 .44     .0003542     .4400000    .0000000
 .48     .0007326     .4800000    .0000000
 .52     .0014361     .5200000   -.0000000
 .56     .0026913     .5600000    .0000000
 .60     .0048549     .6000000   -.0000000
 .64     .0084795     .6400000    .0000000
 .68     .0144114     .6800000   -.0000000
 .72     .0239405     .7200000   -.0000000
 .76     .0390405     .7600000    .0000000
 .80     .0627720     .8000000   -.0000000
 .84     .1000251     .8400000    .0000000
 .88     .1590616     .8800000   -.0000000
 .92     .2554269     .9200000   -.0000000
 .96     .4262064     .9600000   -.0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.5 AND B =     .5
   X      BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000000 NOT DEFINED NOT DEFINED
 .16     .0000000 NOT DEFINED NOT DEFINED
 .20     .0000002     .2000000   -.0000000
 .24     .0000012     .2400000    .0000000
 .28     .0000044     .2800000   -.0000000
 .32     .0000140     .3200000    .0000000
 .36     .0000392     .3600000    .0000000
 .40     .0000988     .4000000   -.0000000
 .44     .0002286     .4400000    .0000000
 .48     .0004940     .4800000    .0000000
 .52     .0010084     .5200001   -.0000001
 .56     .0019616     .5599999    .0000001
 .60     .0036641     .6000000   -.0000000
 .64     .0066125     .6400000    .0000000
 .68     .0115903     .6800000   -.0000000
 .72     .0198247     .7200000   -.0000000
 .76     .0332395     .7600000    .0000000
 .80     .0548836     .8000000   -.0000000
 .84     .0897189     .8400000    .0000000
 .88     .1462538     .8800000   -.0000000
 .92     .2406614     .9200000   -.0000000
 .96     .4116811     .9600000   -.0000000
1.00    1.0000000    1.0000000   0.0000000

A =    9.0 AND B =     .5
   X      BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000000 NOT DEFINED NOT DEFINED
 .16     .0000000 NOT DEFINED NOT DEFINED
 .20     .0000001     .2000000   -.0000000
 .24     .0000006     .2400000    .0000000
 .28     .0000023     .2800000    .0000000
 .32     .0000077     .3200000    .0000000
 .36     .0000229     .3600000    .0000000
 .40     .0000608     .4000000   -.0000000
 .44     .0001477     .4400000    .0000000
 .48     .0003335     .4800000    .0000000
 .52     .0007088     .5200000    .0000000
 .56     .0014314     .5600000   -.0000000
 .60     .0027688     .6000000   -.0000000
 .64     .0051629     .6400000    .0000000
 .68     .0093327     .6800000   -.0000000
 .72     .0164358     .7200000   -.0000000
 .76     .0283323     .7600000    .0000000
 .80     .0480375     .8000000   -.0000000
 .84     .0805539     .8400000    .0000000
 .88     .1345963     .8800000   -.0000000
 .92     .2269203     .9200000   -.0000000
 .96     .3978730     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     1.0 AND B =     1.0
   X       BETA(X)        INVERSE XC   ERROR TERM
 .04      .0400000       .0400000     -.0000000
 .08      .0800000       .0800000     -.0000000
 .12      .1200000       .1200000      .0000000
 .16      .1600000       .1600000      .0000000
 .20      .2000000       .2000000      .0000000
 .24      .2400000       .2400000      .0000000
 .28      .2800000       .2800000      .0000000
 .32      .3200000       .3200000     -.0000000
 .36      .3600000       .3600000     -.0000000
 .40      .4000000       .4000000     -.0000000
 .44      .4400000       .4400000     -.0000000
 .48      .4800000       .4800000     -.0000000
 .52      .5200000       .5200000     -.0000000
 .56      .5600000       .5600000      .0000000
 .60      .6000000       .6000000      .0000000
 .64      .6400000       .6400000      .0000000
 .68      .6800000       .6800000      .0000000
 .72      .7200000       .7200000     -.0000000
 .76      .7600000       .7600000      .0000000
 .80      .8000000       .8000000     -.0000000
 .84      .8400000       .8400000      .0000000
 .88      .8800000       .8800000     -.0000000
 .92      .9200000       .9200000      .0000000
 .96      .9600000       .9600000      .0000000
1.00     1.0000000      1.0000000     0.0000000

A =     1.5 AND B =     1.0
   X       BETA(X)        INVERSE XC   ERROR TERM
 .04      .0080000       .0400000     -.0000000
 .08      .0226274       .0800000     -.0000000
 .12      .0415692       .1199999      .0000001
 .16      .0640000       .1600000     -.0000000
 .20      .0894427       .2000000     -.0000000
 .24      .1175755       .2400000     -.0000000
 .28      .1481621       .2800000     -.0000000
 .32      .1810193       .3200000      .0000000
 .36      .2160000       .3600000     -.0000000
 .40      .2529822       .4000000     -.0000000
 .44      .2918630       .4400000      .0000000
 .48      .3325538       .4800000     -.0000000
 .52      .3749773       .5200000      .0000000
 .56      .4190656       .5600000      .0000000
 .60      .4647580       .6000000     -.0000000
 .64      .5120000       .6400000      .0000000
 .68      .5607424       .6799999      .0000001
 .72      .6109403       .7200000      .0000000
 .76      .6625526       .7600000      .0000000
 .80      .7155418       .8000000     -.0000000
 .84      .7698727       .8400000      .0000000
 .88      .8255132       .8800000      .0000000
 .92      .8824330       .9200000      .0000000
 .96      .9406041       .9600000      .0000000
1.00     1.0000000      1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    2.0 AND B =    1.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04     .0016000      .0400000    -.0000000
 .08     .0064000      .0800000    -.0000000
 .12     .0144000      .1200000    -.0000000
 .16     .0256000      .1600000     .0000000
 .20     .0400000      .2000000    -.0000000
 .24     .0576000      .2400000    -.0000000
 .28     .0784000      .2800000     .0000000
 .32     .1024000      .3200000     .0000000
 .36     .1296000      .3600000    -.0000000
 .40     .1600000      .4000000    -.0000000
 .44     .1936000      .4400000    -.0000000
 .48     .2304000      .4800000    -.0000000
 .52     .2704000      .5200000     .0000000
 .56     .3136000      .5600000    -.0000000
 .60     .3600000      .6000000    -.0000000
 .64     .4096000      .6400000     .0000000
 .68     .4624000      .6800000    -.0000000
 .72     .5184000      .7200000     .0000000
 .76     .5776000      .7600000    -.0000000
 .80     .6400000      .8000000    -.0000000
 .84     .7056000      .8400000     .0000000
 .88     .7744000      .8800000    -.0000000
 .92     .8464000      .9200000     .0000000
 .96     .9216000      .9599999     .0000001
1.00    1.0000000     1.0000000    0.0000000

A =    2.5 AND B =    1.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04     .0003200      .0400000    -.0000000
 .08     .0018102      .0800001    -.0000001
 .12     .0049883      .1200000    -.0000000
 .16     .0102400      .1600000     .0000000
 .20     .0178885      .2000000    -.0000000
 .24     .0282181      .2400000    -.0000000
 .28     .0414854      .2800000     .0000000
 .32     .0579262      .3200000    -.0000000
 .36     .0777600      .3600001    -.0000001
 .40     .1011929      .4000000    -.0000000
 .44     .1284197      .4400000    -.0000000
 .48     .1596258      .4800001    -.0000001
 .52     .1949882      .5200000     .0000000
 .56     .2346768      .5600000    -.0000000
 .60     .2788548      .6000000    -.0000000
 .64     .3276800      .6400000    -.0000000
 .68     .3813048      .6800000    -.0000000
 .72     .4398770      .7200000     .0000000
 .76     .5035400      .7600000    -.0000000
 .80     .5724334      .8000000    -.0000000
 .84     .6466931      .8399999     .0000001
 .88     .7264516      .8800000    -.0000000
 .92     .8118384      .9200000     .0000000
 .96     .9029799      .9600000    -.0000000
1.00    1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     3.0 AND B =     1.0
   X        BETA(X)         INVERSE XC   ERROR TERM
 .04       .0000640       .0400000     -.0000000
 .08       .0005120       .0800000     -.0000000
 .12       .0017280       .1200000      .0000000
 .16       .0040960       .1600000      .0000000
 .20       .0080000       .2000000     -.0000000
 .24       .0138240       .2400000      .0000000
 .28       .0219520       .2800000      .0000000
 .32       .0327680       .3200000     -.0000000
 .36       .0466560       .3600000      .0000000
 .40       .0640000       .4000000     -.0000000
 .44       .0851840       .4400000     -.0000000
 .48       .1105920       .4800000      .0000000
 .52       .1406080       .5200000      .0000000
 .56       .1756160       .5600000     -.0000000
 .60       .2160000       .6000000     -.0000000
 .64       .2621440       .6400000     -.0000000
 .68       .3144320       .6800000     -.0000000
 .72       .3732480       .7200000      .0000000
 .76       .4389760       .7600000     -.0000000
 .80       .5120000       .8000000     -.0000000
 .84       .5927040       .8400000     -.0000000
 .88       .6814720       .8800000     -.0000000
 .92       .7786880       .9200000      .0000000
 .96       .8847360       .9600000     -.0000000
1.00      1.0000000      1.0000000     0.0000000

A =     3.5 AND B =     1.0
   X        BETA(X)         INVERSE XC   ERROR TERM
 .04       .0000128       .0400000     -.0000000
 .08       .0001448       .0800000     -.0000000
 .12       .0005986       .1200000      .0000000
 .16       .0016384       .1600000     -.0000000
 .20       .0035777       .2000000     -.0000000
 .24       .0067723       .2400000      .0000000
 .28       .0116159       .2800000      .0000000
 .32       .0185364       .3200000     -.0000000
 .36       .0279936       .3600000      .0000000
 .40       .0404772       .4000000     -.0000000
 .44       .0565047       .4400000     -.0000000
 .48       .0766204       .4800000      .0000000
 .52       .1013939       .5200000     -.0000000
 .56       .1314190       .5600001     -.0000001
 .60       .1673129       .6000000     -.0000000
 .64       .2097152       .6400000     -.0000000
 .68       .2592873       .6800000      .0000000
 .72       .3167114       .7200000      .0000000
 .76       .3826904       .7600000     -.0000000
 .80       .4579467       .8000000     -.0000000
 .84       .5432222       .8400000     -.0000000
 .88       .6392774       .8800000     -.0000000
 .92       .7468913       .9200000      .0000000
 .96       .8668607-      .9600000     -.0000000
1.00      1.0000000      1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    4.0 AND B =    1.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000026      .0400000     -.0000000
 .08     .0000410      .0800000     -.0000000
 .12     .0002074      .1200000      .0000000
 .16     .0006554      .1600000      .0000000
 .20     .0016000      .2000000     -.0000000
 .24     .0033178      .2400000      .0000000
 .28     .0061466      .2800000     -.0000000
 .32     .0104858      .3200000     -.0000000
 .36     .0167962      .3600000      .0000000
 .40     .0256000      .4000000     -.0000000
 .44     .0374810      .4400001     -.0000001
 .48     .0530842      .4800000      .0000000
 .52     .0731162      .5200000     -.0000000
 .56     .0983450      .5600000      .0000000
 .60     .1296000      .6000000     -.0000000
 .64     .1677722      .6400000     -.0000000
 .68     .2138138      .6800000      .0000000
 .72     .2687386      .7199999      .0000001
 .76     .3336218      .7600000     -.0000000
 .80     .4096000      .8000000     -.0000000
 .84     .4978714      .8400000     -.0000000
 .88     .5996954      .8800000      .0000000
 .92     .7163930      .9200000      .0000000
 .96     .8493466      .9600000     -.0000000
1.00    1.0000000     1.0000000     0.0000000

A =    4.5 AND B =    1.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000005      .0400000     -.0000000
 .08     .0000116      .0800000     -.0000000
 .12     .0000718      .1199999      .0000001
 .16     .0002621      .1600000      .0000000
 .20     .0007155      .2000000     -.0000000
 .24     .0016254      .2400000      .0000000
 .28     .0032525      .2800000     -.0000000
 .32     .0059316      .3200000     -.0000000
 .36     .0100777      .3600000      .0000000
 .40     .0161909      .4000000     -.0000000
 .44     .0248621      .4400000      .0000000
 .48     .0367778      .4800000      .0000000
 .52     .0527248      .5200000     -.0000000
 .56     .0735946      .5600000      .0000000
 .60     .1003877      .6000000     -.0000000
 .64     .1342177      .6400000     -.0000000
 .68     .1763153      .6800000      .0000000
 .72     .2280322      .7200000     -.0000000
 .76     .2908447      .7600000      .0000000
 .80     .3663574      .8000000     -.0000000
 .84     .4563066      .8400000     -.0000000
 .88     .5625641      .8800000      .0000000
 .92     .6871400      .9200000      .0000000
 .96     .8321863      .9600000     -.0000000
1.00    1.0000000     1.0000000     0.0000000
```

## INCOMPLETE BETA FUNCTION AND INVERSE

```
A =     5.0 AND B =    1.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04      .0000001      .0400000    -.0000000
 .08      .0000033      .0800000    -.0000000
 .12      .0000249      .1200000     .0000000
 .16      .0001049      .1600000     .0000000
 .20      .0003200      .2000000    -.0000000
 .24      .0007963      .2400000     .0000000
 .28      .0017210      .2800000     .0000000
 .32      .0033554      .3200000    -.0000000
 .36      .0060466      .3600000     .0000000
 .40      .0102400      .4000000    -.0000000
 .44      .0164916      .4400000     .0000000
 .48      .0254804      .4800000     .0000000
 .52      .0380204      .5200000    -.0000000
 .56      .0550732      .5600000     .0000000
 .60      .0777600      .6000000    -.0000000
 .64      .1073742      .6400000    -.0000000
 .68      .1453934      .6800000     .0000000
 .72      .1934918      .7200000    -.0000000
 .76      .2535525      .7600000     .0000000
 .80      .3276800      .8000000    -.0000000
 .84      .4182119      .8400000    -.0000000
 .88      .5277319      .8800000     .0000000
 .92      .6590815      .9199999     .0000001
 .96      .8153727      .9600000    -.0000000
1.00     1.0000000     1.0000000    0.0000000

A =     5.5 AND B =    1.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000009      .0800000    -.0000000
 .12      .0000086      .1200000     .0000000
 .16      .0000419      .1600000    -.0000000
 .20      .0001431      .2000000    -.0000000
 .24      .0003901      .2400000     .0000000
 .28      .0009107      .2800000     .0000000
 .32      .0018981      .3200000    -.0000000
 .36      .0036280      .3600000     .0000000
 .40      .0064763      .4000000    -.0000000
 .44      .0109393      .4400000     .0000000
 .48      .0176533      .4800000     .0000000
 .52      .0274169      .5200000    -.0000000
 .56      .0412130      .5600000     .0000000
 .60      .0602326      .6000000    -.0000000
 .64      .0858993      .6400001    -.0000001
 .68      .1198944      .6800000     .0000000
 .72      .1641832      .7200000    -.0000000
 .76      .2210420      .7600000     .0000000
 .80      .2930859      .8000000    -.0000000
 .84      .3832976      .8400000    -.0000000
 .88      .4950564      .8800000     .0000000
 .92      .6321688      .9200000    -.0000000
 .96      .7988988      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

 A =     6.0 AND B =     1.0
   X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000003      .0800000     -.0000000
 .12     .0000030      .1200000      .0000000
 .16     .0000168      .1600000      .0000000
 .20     .0000640      .2000000     -.0000000
 .24     .0001911      .2400000      .0000000
 .28     .0004819      .2800000     -.0000000
 .32     .0010737      .3200001     -.0000001
 .36     .0021768      .3599999      .0000001
 .40     .0040960      .4000000     -.0000000
 .44     .0072563      .4400000      .0000000
 .48     .0122306      .4799999      .0000001
 .52     .0197706      .5200000     -.0000000
 .56     .0308410      .5600000      .0000000
 .60     .0466560      .6000000     -.0000000
 .64     .0687195      .6400000      .0000000
 .68     .0988675      .6800000      .0000000
 .72     .1393141      .7200000     -.0000000
 .76     .1926999      .7600000      .0000000
 .80     .2621440      .8000000     -.0000000
 .84     .3512980      .8400000     -.0000000
 .88     .4644041      .8800000      .0000000
 .92     .6063550      .9200000     -.0000000
 .96     .7827578      .9600000      .0000000
1.00    1.0000000     1.0000000     0.0000000

 A =     6.5 AND B =     1.0
   X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000001 NOT DEFINED NOT DEFINED
 .12     .0000010      .1200000     -.0000000
 .16     .0000067      .1600000     -.0000000
 .20     .0000286      .2000000     -.0000000
 .24     .0000936      .2400000     -.0000000
 .28     .0002550      .2800000      .0000000
 .32     .0006074      .3200000      .0000000
 .36     .0013061      .3600000     -.0000000
 .40     .0025905      .4000000     -.0000000
 .44     .0048133      .4400000      .0000000
 .48     .0084736      .4800000     -.0000000
 .52     .0142568      .5200000     -.0000000
 .56     .0230793      .5600000      .0000000
 .60     .0361396      .6000000     -.0000000
 .64     .0549756      .6400000      .0000000
 .68     .0815282      .6800000      .0000000
 .72     .1182119      .7200000     -.0000000
 .76     .1679919      .7600000      .0000000
 .80     .2344687      .8000000     -.0000000
 .84     .3219700      .8400000     -.0000000
 .88     .4356496      .8800000      .0000000
 .92     .5815953      .9200000     -.0000000
 .96     .7669429      .9600000      .0000000
1.00    1.0000000     1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    7.0 AND B =    1.0
   X       BETA(X)        INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000000 NOT DEFINED NOT DEFINED
 .12      .0000004    .1200000   -.0000000
 .16      .0000027    .1600000   -.0000000
 .20      .0000128    .2000000   -.0000000
 .24      .0000459    .2400000   -.0000000
 .28      .0001349    .2800000   -.0000000
 .32      .0003436    .3200000    .0000000
 .36      .0007836    .3600000   -.0000000
 .40      .0016384    .4000000   -.0000000
 .44      .0031928    .4400000    .0000000
 .48      .0058707    .4800000   -.0000000
 .52      .0102807    .5200000   -.0000000
 .56      .0172709    .5600000    .0000000
 .60      .0279936    .6000000   -.0000000
 .64      .0439805    .6400000    .0000000
 .68      .0672299    .6800000    .0000000
 .72      .1003061    .7200000   -.0000000
 .76      .1464519    .7600000    .0000000
 .80      .2097152    .8000000   -.0000000
 .84      .2950903    .8400001   -.0000001
 .88      .4086756    .8799999    .0000001
 .92      .5578466    .9200000   -.0000000
 .96      .7514475    .9600000    .0000000
1.00     1.0000000   1.0000000   0.0000000

A =    7.5 AND B =    1.0
   X       BETA(X)        INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000000 NOT DEFINED NOT DEFINED
 .12      .0000001    .1200000   -.0000000
 .16      .0000011    .1600000   -.0000000
 .20      .0000057    .2000000   -.0000000
 .24      .0000225    .2400000   -.0000000
 .28      .0000714    .2800000   -.0000000
 .32      .0001944    .3200000    .0000000
 .36      .0004702    .3600001   -.0000001
 .40      .0010362    .4000000   -.0000000
 .44      .0021178    .4399999    .0000001
 .48      .0040673    .4800001   -.0000001
 .52      .0074135    .5200000   -.0000000
 .56      .0129244    .5600000    .0000000
 .60      .0216837    .6000000   -.0000000
 .64      .0351844    .6400000    .0000000
 .68      .0554392    .6800000    .0000000
 .72      .0851126    .7200000   -.0000000
 .76      .1276738    .7600000    .0000000
 .80      .1875750    .8000000   -.0000000
 .84      .2704548    .8400000    .0000000
 .88      .3833717    .8800000    .0000000
 .92      .5350677    .9200000   -.0000000
 .96      .7362652    .9600000    .0000000
1.00     1.0000000   1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     8.0 AND B =     1.0
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000000 NOT DEFINED NOT DEFINED
 .12      .0000000 NOT DEFINED NOT DEFINED
 .16      .0000004     .1600000     .0000000
 .20      .0000026     .2000000    -.0000000
 .24      .0000110     .2400000     .0000000
 .28      .0000378     .2800000     .0000000
 .32      .0001100     .3200000     .0000000
 .36      .0002821     .3600000     .0000000
 .40      .0006554     .4000000    -.0000000
 .44      .0014048     .4400000     .0000000
 .48      .0028179     .4800000     .0000000
 .52      .0053460     .5200000    -.0000000
 .56      .0096717     .5600000     .0000000
 .60      .0167962     .6000000    -.0000000
 .64      .0281475     .6400000     .0000000
 .68      .0457163     .6800000     .0000000
 .72      .0722204     .7200000    -.0000000
 .76      .1113035     .7600000     .0000000
 .80      .1677722     .8000000    -.0000000
 .84      .2478759     .8400000     .0000000
 .88      .3596345     .8800000     .0000000
 .92      .5132189     .9200000    -.0000000
 .96      .7213896     .9600000     .0000000
1.00     1.0000000    1.0000000    0.0000000

A =     8.5 AND B =     1.0
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000000 NOT DEFINED NOT DEFINED
 .12      .0000000 NOT DEFINED NOT DEFINED
 .16      .0000002     .1599999     .0000001
 .20      .0000011     .2000000    -.0000000
 .24      .0000054     .2400000     .0000000
 .28      .0000200     .2799999     .0000001
 .32      .0000622     .3200000     .0000000
 .36      .0001693     .3600000     .0000000
 .40      .0004145     .4000000    -.0000000
 .44      .0009319     .4400000     .0000000
 .48      .0019523     .4800000     .0000000
 .52      .0038550     .5200000    -.0000000
 .56      .0072377     .5600000     .0000000
 .60      .0130102     .6000000    -.0000000
 .64      .0225180     .6400000     .0000000
 .68      .0376986     .6800000    -.0000000
 .72      .0612811     .7200000    -.0000000
 .76      .0970321     .7600000     .0000000
 .80      .1500600     .8000000    -.0000000
 .84      .2271820     .8400000     .0000000
 .88      .3373671     .8800000     .0000000
 .92      .4922623     .9200000    -.0000000
 .96      .7068145     .9600000     .0000000
1.00     1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    9.0 AND B =    1.0
  X       BETA(X)        INVERSE XC  ERROR TERM
.04     .0000000 NOT DEFINED NOT DEFINED
.08     .0000000 NOT DEFINED NOT DEFINED
.12     .0000000 NOT DEFINED NOT DEFINED
.16     .0000001 NOT DEFINED NOT DEFINED
.20     .0000005     .2000000   -.0000000
.24     .0000026     .2400000    .0000000
.28     .0000106     .2800000    .0000000
.32     .0000352     .3200000    .0000000
.36     .0001016     .3600000    .0000000
.40     .0002621     .4000000   -.0000000
.44     .0006181     .4400000    .0000000
.48     .0013526     .4800000    .0000000
.52     .0027799     .5200001   -.0000001
.56     .0054162     .5599999    .0000001
.60     .0100777     .6000000   -.0000000
.64     .0180144     .6400000    .0000000
.68     .0310871     .6800000   -.0000000
.72     .0519987     .7200000   -.0000000
.76     .0845906     .7600000    .0000000
.80     .1342177     .8000000   -.0000000
.84     .2082157     .8400000    .0000000
.88     .3164784     .8800000    .0000000
.92     .4721614     .9200000   -.0000000
.96     .6925340     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000

A =    1.5 AND B =    1.5
  X       BETA(X)        INVERSE XC  ERROR TERM
.04     .0134171     .0400000   -.0000000
.08     .0374780     .0800000   -.0000000
.12     .0679724     .1199999    .0000001
.16     .1032755     .1600000   -.0000000
.20     .1423785     .2000000   -.0000000
.24     .1845494     .2400000    .0000000
.28     .2292082     .2800000   -.0000000
.32     .2758683     .3200000    .0000000
.36     .3241039     .3600000    .0000000
.40     .3735301     .4000000   -.0000000
.44     .4237894     .4400000    .0000000
.48     .4745420     .4800000    .0000000
.52     .5254580     .5200000    .0000000
.56     .5762106     .5600000    .0000000
.60     .6264699     .6000000    .0000000
.64     .6758961     .6400000    .0000000
.68     .7241317     .6800000    .0000000
.72     .7707918     .7200000    .0000000
.76     .8154506     .7599999    .0000001
.80     .8576215     .8000000    .0000000
.84     .8967245     .8400000    .0000000
.88     .9320276     .8800000    .0000000
.92     .9625220     .9200000    .0000000
.96     .9865829     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

  A =     2.0 AND B =     1.5
    X       BETA(X)        INVERSE XC   ERROR TERM
  .04      .0029597      .0400000     -.0000000
  .08      .0116750      .0800000     -.0000000
  .12      .0258945      .1200000     -.0000000
  .16      .0453578      .1600000      .0000000
  .20      .0697957      .2000000     -.0000000
  .24      .0989284      .2400000     -.0000000
  .28      .1324648      .2800001     -.0000001
  .32      .1701013      .3200000      .0000000
  .36      .2115200      .3600000     -.0000000
  .40      .2563872      .4000000     -.0000000
  .44      .3043511      .4400000      .0000000
  .48      .3550390      .4800000     -.0000000
  .52      .4080543      .5200000      .0000000
  .56      .4629721      .5600000      .0000000
  .60      .5193338      .6000000     -.0000000
  .64      .5766400      .6400000      .0000000
  .68      .6343409      .6800000     -.0000000
  .72      .6918229      .7200000      .0000000
  .76      .7483884      .7600000      .0000000
  .80      .8032260      .8000000      .0000000
  .84      .8553600      .8400000      .0000000
  .88      .9035594      .8800000      .0000000
  .92      .9461467      .9199999      .0000001
  .96      .9804800      .9600000      .0000000
 1.00     1.0000000     1.0000000     0.0000000

  A =     2.5 AND B =     1.5
    X       BETA(X)        INVERSE XC   ERROR TERM
  .04      .0006425      .0400000     -.0000000
  .08      .0035806      .0800000     -.0000000
  .12      .0097159      .1200000     -.0000000
  .16      .0196290      .1600000      .0000000
  .20      .0337287      .2000000     -.0000000
  .24      .0523023      .2400000     -.0000000
  .28      .0755397      .2800000      .0000000
  .32      .1035476      .3199999      .0000001
  .36      .1363570      .3600000     -.0000000
  .40      .1739276      .4000000     -.0000000
  .44      .2161499      .4400000     -.0000000
  .48      .2628445      .4800000     -.0000000
  .52      .3137606      .5200000      .0000000
  .56      .3685713      .5600000     -.0000000
  .60      .4268677      .6000000     -.0000000
  .64      .4881494      .6400000      .0000000
  .68      .5518112      .6800000     -.0000000
  .72      .6171235      .7200000      .0000000
  .76      .6832036      .7600000     -.0000000
  .80      .7489718      .8000000      .0000000
  .84      .8130781      .8400000      .0000000
  .88      .8737710      .8800000      .0000000
  .92      .9286247      .9200000      .0000000
  .96      .9738084      .9600000      .0000000
 1.00     1.0000000     1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    3.0 AND B =    1.5
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04     .0001379     .0400001   -.0000001
 .08     .0010858     .0800000   -.0000000
 .12     .0036056     .1200000    .0000000
 .16     .0084039     .1600000    .0000000
 .20     .0161301     .2000000   -.0000000
 .24     .0273727     .2400000    .0000000
 .28     .0426566     .2799999    .0000001
 .32     .0624388     .3200000   -.0000000
 .36     .0871040     .3600000    .0000000
 .40     .1169598     .4000000   -.0000000
 .44     .1522302     .4400000   -.0000000
 .48     .1930488     .4800000    .0000000
 .52     .2394496     .5200000    .0000000
 .56     .2913567     .5600000   -.0000000
 .60     .3485708     .6000000   -.0000000
 .64     .4107520     .6399999    .0000001
 .68     .4773972     .6800000   -.0000000
 .72     .5478094     .7200000    .0000000
 .76     .6210541     .7600000    .0000000
 .80     .6958948     .8000000    .0000000
 .84     .7706880     .8400000    .0000000
 .88     .8432009     .8800000    .0000000
 .92     .9102370     .9200000    .0000000
 .96     .9666560     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000

A =    3.5 AND B =    1.5
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000293     .0400000   -.0000000
 .08     .0003265     .0800000   -.0000000
 .12     .0013269     .1200000    .0000000
 .16     .0035688     .1600000   -.0000000
 .20     .0076528     .2000000   -.0000000
 .24     .0142151     .2400000    .0000000
 .28     .0239071     .2800000    .0000000
 .32     .0373765     .3200000   -.0000000
 .36     .0552504     .3600000    .0000000
 .40     .0781185     .4000000   -.0000000
 .44     .1065162     .4400000   -.0000000
 .48     .1409067     .4800000    .0000000
 .52     .1816614     .5200000    .0000000
 .56     .2290376     .5600000   -.0000000
 .60     .2831540     .6000000   -.0000000
 .64     .3439599     .6400000   -.0000000
 .68     .4111977     .6800000   -.0000000
 .72     .4843540     .7200000    .0000000
 .76     .5625943     .7600000   -.0000000
 .80     .6446680     .8000000   -.0000000
 .84     .7287624     .8400000   -.0000000
 .88     .8122521     .8800000    .0000000
 .92     .8912020     .9200000    .0000000
 .96     .9590921     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     4.0 AND B =     1.5
   X       BETA(X)          INVERSE  XC   ERROR TERM
 .04      .0000062        .0400000     -.0000000
 .08      .0000975        .0800000     -.0000000
 .12      .0004852        .1200000      .0000000
 .16      .0015059        .1600000      .0000000
 .20      .0036081        .2000000     -.0000000
 .24      .0073371        .2400000      .0000000
 .28      .0133193        .2799999      .0000001
 .32      .0222448        .3200000     -.0000000
 .36      .0348493        .3600000      .0000000
 .40      .0518937        .4000000     -.0000000
 .44      .0741415        .4400000     -.0000000
 .48      .1023343        .4800000      .0000000
 .52      .1371627        .5200000     -.0000000
 .56      .1792346        .5600001     -.0000001
 .60      .2290367        .6000000     -.0000000
 .64      .2868890        .6400000     -.0000000
 .68      .3528885        .6800001     -.0000001
 .72      .4268380        .7200000      .0000000
 .76      .5081511        .7600000     -.0000000
 .80      .5957189        .8000000     -.0000000
 .84      .6877094        .8400000      .0000000
 .88      .7812328        .8800000     -.0000000
 .92      .8716939        .9200000      .0000000
 .96      .9511731        .9599999      .0000001
1.00     1.0000000       1.0000000     0.0000000

A =     4.5 AND B =     1.5
   X       BETA(X)          INVERSE  XC   ERROR TERM
 .04      .0000013        .0400000     -.0000000
 .08      .0000290        .0800000     -.0000000
 .12      .0001765        .1200000      .0000000
 .16      .0006321        .1600000      .0000000
 .20      .0016926        .2000000     -.0000000
 .24      .0037684        .2399999      .0000001
 .28      .0073847        .2800000     -.0000000
 .32      .0131768        .3200000     -.0000000
 .36      .0218808        .3599999      .0000001
 .40      .0343200        .4000000     -.0000000
 .44      .0513862        .4400001     -.0000001
 .48      .0740152        .4800000      .0000000
 .52      .1031567        .5200000     -.0000000
 .56      .1397360        .5600000      .0000000
 .60      .1846074        .6000000     -.0000000
 .64      .2384956        .6400000     -.0000000
 .68      .3019208        .6800000      .0000000
 .72      .3751037        .7200000      .0000000
 .76      .4578365        .7600000     -.0000000
 .80      .5493046        .8000000     -.0000000
 .84      .6478194        .8400000      .0000000
 .88      .7503817        .8800001     -.0000001
 .92      .8518548        .9200000      .0000000
 .96      .9429462        .9600000      .0000000
1.00     1.0000000       1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

 A =    5.0 AND B =    1.5
   X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000003     .0400000    -.0000000
 .08     .0000086     .0800000    -.0000000
 .12     .0000639     .1200000     .0000000
 .16     .0002642     .1600000     .0000000
 .20     .0007907     .2000000    -.0000000
 .24     .0019275     .2400000     .0000000
 .28     .0040780     .2800000     .0000000
 .32     .0077749     .3200000    -.0000000
 .36     .0136861     .3600000     .0000000
 .40     .0226139     .4000000    -.0000000
 .44     .0354876     .4400000     .0000000
 .48     .0533484     .4800000     .0000000
 .52     .0773249     .5200000    -.0000000
 .56     .1085977     .5600000     .0000000
 .60     .1483512     .6000000    -.0000000
 .64     .1977076     .6400000    -.0000000
 .68     .2576393     .6800000     .0000000
 .72     .3288512     .7200000     .0000000
 .76     .4116190     .7600000    -.0000000
 .80     .5055606     .8000000    -.0000000
 .84     .6092947     .8400000    -.0000000
 .88     .7198845     .8800000    -.0000000
 .92     .8318018     .9200000     .0000000
 .96     .9344516     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000

 A =    5.5 AND B =    1.5
   X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000001 NOT DEFINED NOT DEFINED
 .08     .0000025     .0800000    -.0000000
 .12     .0000231     .1200000     .0000000
 .16     .0001101     .1599999     .0000001
 .20     .0003681     .2000000    -.0000000
 .24     .0009825     .2400000     .0000000
 .28     .0022444     .2799999     .0000001
 .32     .0045725     .3200000    -.0000000
 .36     .0085330     .3600000     .0000000
 .40     .0148541     .4000000    -.0000000
 .44     .0244337     .4400000     .0000000
 .48     .0383397     .4800000     .0000000
 .52     .0577985     .5200000    -.0000000
 .56     .0841706     .5600000     .0000000
 .60     .1189098     .6000000    -.0000000
 .64     .1634987     .6400000    -.0000000
 .68     .2193561     .6800000     .0000000
 .72     .2877034     .7199999     .0000001
 .76     .3693743     .7600000    -.0000000
 .80     .4645371     .8000000    -.0000000
 .84     .5722726     .8400000    -.0000000
 .88     .6898862     .8800000    -.0000000
 .92     .8116332     .9200000     .0000000
 .96     .9257240     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.0 AND B =    1.5
   X      BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000007      .0800000   -.0000000
 .12     .0000083      .1200000    .0000000
 .16     .0000457      .1600000    .0000000
 .20     .0001708      .2000000   -.0000000
 .24     .0004994      .2400000    .0000000
 .28     .0012317      .2799999    .0000001
 .32     .0026815      .3200001   -.0000001
 .36     .0053055      .3600000    .0000000
 .40     .0097308      .4000000   -.0000000
 .44     .0167792      .4400000    .0000000
 .48     .0274839      .4800000    .0000000
 .52     .0430976      .5200000   -.0000000
 .56     .0650853      .5600000    .0000000
 .60     .0950988      .6000000   -.0000000
 .64     .1349239      .6400000   -.0000000
 .68     .1863929      .6800000    .0000000
 .72     .2512456      .7200000   -.0000000
 .76     .3309181      .7600000    .0000000
 .80     .4262214      .8000000   -.0000000
 .84     .5368395      .8400000   -.0000000
 .88     .6604992      .8800000   -.0000000
 .92     .7914310      .9200001   -.0000001
 .96     .9167937      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000

A =    6.5 AND B =    1.5
   X      BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000002      .0800000   -.0000000
 .12     .0000030      .1200000   -.0000000
 .16     .0000189      .1599999    .0000001
 .20     .0000791      .2000000   -.0000000
 .24     .0002532      .2400000   -.0000000
 .28     .0006743      .2800000    .0000000
 .32     .0015688      .3200000    .0000000
 .36     .0032909      .3600000   -.0000000
 .40     .0063598      .4000000   -.0000000
 .44     .0114965      .4400000    .0000000
 .48     .0196587      .4800000   -.0000000
 .52     .0320680      .5200000   -.0000000
 .56     .0502251      .5600000    .0000000
 .60     .0759076      .6000000   -.0000000
 .64     .1111373      .6400001   -.0000001
 .68     .1581081      .6799999    .0000001
 .72     .2190545      .7200000   -.0000000
 .76     .2960312      .7600000    .0000000
 .80     .3905581      .8000000   -.0000000
 .84     .5030442      .8400000   -.0000000
 .88     .6318104      .8800000   -.0000000
 .92     .7712653      .9200000   -.0000000
 .96     .9076876      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     7.0 AND B =    1.5
  X        BETA(X)        INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000001 NOT DEFINED NOT DEFINED
 .12      .0000011     .1200000   -.0000000
 .16      .0000078     .1600000    .0000000
 .20      .0000365     .2000000   -.0000000
 .24      .0001281     .2400000   -.0000000
 .28      .0003683     .2799999    .0000001
 .32      .0009158     .3200000    .0000000
 .36      .0020371     .3600000   -.0000000
 .40      .0041482     .4000000   -.0000000
 .44      .0078614     .4400000    .0000000
 .48      .0140344     .4800000   -.0000000
 .52      .0238163     .5200000   -.0000000
 .56      .0386879     .5600000    .0000000
 .60      .0604847     .6000000   -.0000000
 .64      .0913938     .6400000    .0000000
 .68      .1339081     .6800000    .0000000
 .72      .1907133     .7200000   -.0000000
 .76      .2644744     .7600000    .0000000
 .80      .3574607     .8000000   -.0000000
 .84      .4709052     .8400000   -.0000000
 .88      .6038853     .8800000    .0000000
 .92      .7511948     .9200000    .0000000
 .96      .8984295     .9600000    .0000000
1.00     1.0000000    1.0000000   0.0000000

A =     7.5 AND B =    1.5
  X        BETA(X)        INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000000 NOT DEFINED NOT DEFINED
 .12      .0000004     .1200000   -.0000000
 .16      .0000032     .1600000    .0000000
 .20      .0000168     .2000000   -.0000000
 .24      .0000647     .2400000   -.0000000
 .28      .0002008     .2800000    .0000000
 .32      .0005336     .3200000    .0000000
 .36      .0012586     .3600000   -.0000000
 .40      .0027008     .4000000   -.0000000
 .44      .0053663     .4400000    .0000000
 .48      .0100020     .4800000   -.0000000
 .52      .0176589     .5200000   -.0000000
 .56      .0297534     .5600000    .0000000
 .60      .0481217     .6000000   -.0000000
 .64      .0750481     .6400000    .0000000
 .68      .1132557     .6800000    .0000000
 .72      .1658251     .7200000   -.0000000
 .76      .2360026     .7600000    .0000000
 .80      .3268225     .8000000   -.0000000
 .84      .4404191     .8400000   -.0000000
 .88      .5767725     .8800000    .0000000
 .92      .7312700     .9200000    .0000000
 .96      .8890407     .9600000   -.0000000
1.00     1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.0 AND B =    1.5
   X      BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000001    .1200000     .0000000
 .16     .0000013    .1600000     .0000000
 .20     .0000077    .2000000    -.0000000
 .24     .0000326    .2400000     .0000000
 .28     .0001093    .2800000    -.0000000
 .32     .0003104    .3200000     .0000000
 .36     .0007764    .3600000     .0000000
 .40     .0017556    .4000000    -.0000000
 .44     .0036574    .4400000     .0000000
 .48     .0071174    .4800000     .0000000
 .52     .0130738    .5200000    -.0000000
 .56     .0228494    .5600000     .0000000
 .60     .0382328    .6000000    -.0000000
 .64     .0615447    .6400000     .0000000
 .68     .0956691    .6800000     .0000000
 .72     .1440169    .7200000    -.0000000
 .76     .2103703    .7600000     .0000000
 .80     .2985229    .8000000    -.0000000
 .84     .4115644    .8400000    -.0000000
 .88     .5505064    .8800000     .0000000
 .92     .7115334    .9200000     .0000000
 .96     .8795405    .9600000    -.0000000
1.00    1.0000000   1.0000000    0.0000000

A =    8.5 AND B =    1.5
   X      BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000000 NOT DEFINED NOT DEFINED
 .16     .0000005    .1600000    -.0000000
 .20     .0000036    .2000000    -.0000000
 .24     .0000164    .2400000     .0000000
 .28     .0000594    .2800000     .0000000
 .32     .0001803    .3200000     .0000000
 .36     .0004782    .3600000     .0000000
 .40     .0011396    .4000000    -.0000000
 .44     .0024892    .4400000     .0000000
 .48     .0050578    .4800000     .0000000
 .52     .0096666    .5200000    -.0000000
 .56     .0175250    .5600000     .0000000
 .60     .0303386    .6000000    -.0000000
 .64     .0504113    .6400000     .0000000
 .68     .0807227    .6800000     .0000000
 .72     .1249450    .7200000    -.0000000
 .76     .1873394    .7600000     .0000000
 .80     .2724347    .8000000    -.0000000
 .84     .3843071    .8400000    -.0000000
 .88     .5251103    .8800000     .0000000
 .92     .6920213    .9200000     .0000000
 .96     .8699463    .9600000    -.0000000
1.00    1.0000000   1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

 A =     9.0 AND B =     1.5
   X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT  DEFINED NOT DEFINED
 .08     .0000000 NOT  DEFINED NOT DEFINED
 .12     .0000000 NOT  DEFINED NOT DEFINED
 .16     .0000002      .1600000   -.0000000
 .20     .0000016      .2000000   -.0000000
 .24     .0000082      .2400000    .0000000
 .28     .0000322      .2800000    .0000000
 .32     .0001046      .3200000    .0000000
 .36     .0002942      .3599999    .0000001
 .40     .0007387      .4000000   -.0000000
 .44     .0016920      .4400000    .0000000
 .48     .0035898      .4800000    .0000000
 .52     .0071386      .5200000   -.0000000
 .56     .0134255      .5600000    .0000000
 .60     .0240472      .6000000   -.0000000
 .64     .0412472      .6400000    .0000000
 .68     .0680415      .6800000    .0000000
 .72     .1082942      .7200000   -.0000000
 .76     .1666812      .7600000    .0000000
 .80     .2484259      .8000000   -.0000000
 .84     .3586027      .8400000   -.0000000
 .88     .5005972      .8800000    .0000000
 .92     .6727643      .9200000    .0000000
 .96     .8602738      .9600000   -.0000000
1.00    1.0000000     1.0000000   0.0000000

 A =     2.0 AND B =     2.0
   X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0046720      .0400000   -.0000000
 .08     .0181760      .0800000   -.0000000
 .12     .0397440      .1200000   -.0000000
 .16     .0686080      .1600000    .0000000
 .20     .1040000      .2000000   -.0000000
 .24     .1451520      .2400000   -.0000000
 .28     .1912960      .2800000   -.0000000
 .32     .2416640      .3200000    .0000000
 .36     .2954880      .3600000   -.0000000
 .40     .3520000      .4000000   -.0000000
 .44     .4104320      .4400000    .0000000
 .48     .4700160      .4800000    .0000000
 .52     .5299840      .5200000    .0000000
 .56     .5895680      .5600000    .0000000
 .60     .6480000      .6000000    .0000000
 .64     .7045120      .6400000    .0000000
 .68     .7583360      .6800000    .0000000
 .72     .8087040      .7200000    .0000000
 .76     .8548480      .7600000    .0000000
 .80     .8960000      .8000000    .0000000
 .84     .9313920      .8400000    .0000000
 .88     .9602560      .8800000    .0000000
 .92     .9818240      .9200000    .0000000
 .96     .9953280      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     2.5 AND B =     2.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04      .0010880      .0400000    -.0000000
 .08      .0059736      .0800000    -.0000000
 .12      .0159626      .1200000    -.0000000
 .16      .0317440      .1599999     .0000001
 .20      .0536656      .2000000    -.0000000
 .24      .0818326      .2400000    -.0000000
 .28      .1161591      .2800000     .0000000
 .32      .1564007      .3200000     .0000000
 .36      .2021760      .3600000    -.0000000
 .40      .2529822      .4000000    -.0000000
 .44      .3082073      .4399999     .0000001
 .48      .3671393      .4800000    -.0000000
 .52      .4289741      .5200000     .0000000
 .56      .4928212      .5600000     .0000000
 .60      .5577096      .6000000     .0000000
 .64      .6225920      .6400000     .0000000
 .68      .6863487      .6800000     .0000000
 .72      .7477909      .7200000     .0000000
 .76      .8056640      .7600000     .0000000
 .80      .8586501      .8000000     .0000000
 .84      .9053703      .8400000     .0000000
 .88      .9443871      .8800000     .0000000
 .92      .9742060      .9200000     .0000000
 .96      .9932779      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000

A =     3.0 AND B =     2.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04      .0002483      .0400001    -.0000001
 .08      .0019251      .0800000    -.0000000
 .12      .0062899      .1200000     .0000000
 .16      .0144179      .1600000     .0000000
 .20      .0272000      .2000000    -.0000000
 .24      .0453427      .2400000     .0000000
 .28      .0693683      .2800000     .0000000
 .32      .0996147      .3200000    -.0000000
 .36      .1362355      .3600001    -.0000001
 .40      .1792000      .4000000    -.0000000
 .44      .2282931      .4400000    -.0000000
 .48      .2831155      .4800000    -.0000000
 .52      .3430835      .5200000     .0000000
 .56      .4074291      .5600000    -.0000000
 .60      .4752000      .6000000    -.0000000
 .64      .5452595      .6400000     .0000000
 .68      .6162867      .6800000    -.0000000
 .72      .6867763      .7200000     .0000000
 .76      .7550387      .7599999     .0000001
 .80      .8192000      .8000000     .0000000
 .84      .8772019      .8400000     .0000000
 .88      .9268019      .8800000     .0000000
 .92      .9655731      .9200000     .0000000
 .96      .9909043      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    3.5 AND B =    2.0
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000558     .0400000    -.0000000
 .08    .0006111     .0800000    -.0000000
 .12    .0024423     .1200000     .0000000
 .16    .0064553     .1600000    -.0000000
 .20    .0135953     .2000000    -.0000000
 .24    .0247868     .2400000     .0000000
 .28    .0408880     .2800000     .0000000
 .32    .0626530     .3200000    -.0000000
 .36    .0906993     .3600000     .0000000
 .40    .1254792     .4000000    -.0000000
 .44    .1672538     .4400000    -.0000000
 .48    .2160695     .4800000     .0000000
 .52    .2717356     .5200000     .0000000
 .56    .3338042     .5600000    -.0000000
 .60    .4015509     .6000000    -.0000000
 .64    .4739564     .6400000     .0000000
 .68    .5496890     .6800000    -.0000000
 .72    .6270886     .7200000     .0000000
 .76    .7041503     .7600000     .0000000
 .80    .7785094     .8000000     .0000000
 .84    .8474266     .8400000     .0000000
 .88    .9077739     .8799999     .0000001
 .92    .9560209     .9200000     .0000000
 .96    .9882212     .9600000     .0000000
1.00   1.0000000    1.0000000    0.0000000

A =    4.0 AND B =    2.0
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000124     .0400000    -.0000000
 .08    .0001917     .0800000    -.0000000
 .12    .0009373     .1200000     .0000000
 .16    .0028574     .1600000     .0000000
 .20    .0067200     .2000000    -.0000000
 .24    .0134038     .2400000     .0000000
 .28    .0238487     .2800000     .0000000
 .32    .0390070     .3200000    -.0000000
 .36    .0597943     .3600000     .0000000
 .40    .0870400     .4000000    -.0000000
 .44    .1214383     .4400000    -.0000000
 .48    .1634992     .4800000     .0000000
 .52    .2134992     .5200000     .0000000
 .56    .2714321     .5600000    -.0000000
 .60    .3369600     .6000000    -.0000000
 .64    .4093641     .6400000    -.0000000
 .68    .4874954     .6800000    -.0000000
 .72    .5697257     .7200000     .0000000
 .76    .6538986     .7600000    -.0000000
 .80    .7372800     .8000000     .0000000
 .84    .8165090     .8399999     .0000001
 .88    .8875491     .8800000     .0000000
 .92    .9456387     .9200000     .0000000
 .96    .9852420     .9600000     .0000000
1.00   1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

 A =    4.5 AND B =    2.0
   X       BETA(X)       INVERSE XC  ERROR TERM
  .04     .0000027     .0400000   -.0000000
  .08     .0000595     .0800000   -.0000000
  .12     .0003563     .1200000    .0000000
  .16     .0012530     .1600000    .0000000
  .20     .0032915     .2000000   -.0000000
  .24     .0071841     .2400000    .0000000
  .28     .0137904     .2800000   -.0000000
  .32     .0240825     .3200000   -.0000000
  .36     .0391015     .3600000    .0000000
  .40     .0599062     .4000000   -.0000000
  .44     .0875144     .4400000   -.0000000
  .48     .1228378     .4800000    .0000000
  .52     .1666104     .5200000   -.0000000
  .56     .2193120     .5600001   -.0000001
  .60     .2810856     .6000000   -.0000000
  .64     .3516504     .6400000   -.0000000
  .68     .4302094     .6800000   -.0000000
  .72     .5153528     .7200000    .0000000
  .76     .6049570     .7600000    .0000000
  .80     .6960790     .8000000    .0000000
  .84     .7848474     .8400000    .0000000
  .88     .8663487     .8800000    .0000000
  .92     .9345104     .9200000    .0000000
  .96     .9819798     .9600000    .0000000
 1.00    1.0000000    1.0000000   0.0000000

 A =    5.0 AND B =    2.0
   X       BETA(X)       INVERSE XC  ERROR TERM
  .04     .0000006     .0400000   -.0000000
  .08     .0000184     .0800000   -.0000000
  .12     .0001344     .1200000    .0000000
  .16     .0005453     .1600000   -.0000000
  .20     .0016000     .2000000   -.0000000
  .24     .0038221     .2400000    .0000000
  .28     .0079168     .2800001   -.0000001
  .32     .0147640     .3200000   -.0000000
  .36     .0253958     .3600000    .0000000
  .40     .0409600     .4000000   -.0000000
  .44     .0626682     .4400000    .0000000
  .48     .0917294     .4799999    .0000001
  .52     .1292694     .5200000   -.0000000
  .56     .1762342     .5600000    .0000000
  .60     .2332800     .6000000   -.0000000
  .64     .3006477     .6400000   -.0000000
  .68     .3780227     .6800001   -.0000001
  .72     .4643802     .7200000    .0000000
  .76     .5578156     .7600000   -.0000000
  .80     .6553600     .8000000    .0000000
  .84     .7527815     .8400000    .0000000
  .88     .8443711     .8800000    .0000000
  .92     .9227141     .9200000    .0000000
  .96     .9784472     .9599999    .0000001
 1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    5.5 AND B =    2.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04    .0000001       .0400000   -.0000000
 .08    .0000056       .0800000   -.0000000
 .12    .0000503       .1200000    .0000000
 .16    .0002357       .1600000    .0000000
 .20    .0007728       .2000000   -.0000000
 .24    .0020207       .2400000    .0000000
 .28    .0045170       .2800000    .0000000
 .32    .0089971       .3200000   -.0000000
 .36    .0163984       .3600000    .0000000
 .40    .0278483       .4000000   -.0000000
 .44    .0446324       .4400000    .0000000
 .48    .0681419       .4800000    .0000000
 .52    .0997975       .5200000   -.0000000
 .56    .1409484       .5600000    .0000000
 .60    .1927444       .6000000   -.0000000
 .64    .2559801       .6400000   -.0000000
 .68    .3309086       .6800000    .0000000
 .72    .4170253       .7200000    .0000000
 .76    .5128174       .7600000   -.0000000
 .80    .6154804       .8000000   -.0000000
 .84    .7205994       .8400000    .0000000
 .88    .8217937       .8800000    .0000000
 .92    .9103231       .9200000    .0000000
 .96    .9746566       .9600000    .0000000
1.00   1.0000000      1.0000000   0.0000000

A =    6.0 AND B =    2.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000017       .0800000   -.0000000
 .12    .0000188       .1200000    .0000000
 .16    .0001013       .1600000    .0000000
 .20    .0003712       .2000000   -.0000000
 .24    .0010625       .2400000    .0000000
 .28    .0025637       .2800000    .0000000
 .32    .0054546       .3200000   -.0000000
 .36    .0105356       .3600000    .0000000
 .40    .0188416       .4000000   -.0000000
 .44    .0316375       .4400000    .0000000
 .48    .0503900       .4800000    .0000000
 .52    .0767100       .5200000   -.0000000
 .56    .1122612       .5600000    .0000000
 .60    .1586304       .6000000   -.0000000
 .64    .2171535       .6400000   -.0000000
 .68    .2886930       .6800000    .0000000
 .72    .3733617       .7200000    .0000000
 .76    .4701878       .7600000   -.0000000
 .80    .5767168       .8000000   -.0000000
 .84    .6885441       .8400000   -.0000000
 .88    .7987750       .8800000    .0000000
 .92    .8974054       .9200000    .0000000
 .96    .9706197       .9600000    .0000000
1.00   1.0000000      1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.5 AND B =    2.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000005     .0800000   -.0000000
 .12     .0000070     .1199999    .0000001
 .16     .0000434     .1600000    .0000000
 .20     .0001775     .2000000   -.0000000
 .24     .0005561     .2399999    .0000001
 .28     .0014484     .2800000   -.0000000
 .32     .0032921     .3200001   -.0000001
 .36     .0067393     .3599999    .0000001
 .40     .0126936     .4000000   -.0000000
 .44     .0223337     .4400000    .0000000
 .48     .0371144     .4800000    .0000000
 .52     .0587380     .5200000   -.0000000
 .56     .0890860     .5600000    .0000000
 .60     .1301025     .6000000   -.0000000
 .64     .1836184     .6400000   -.0000000
 .68     .2511069     .6800000    .0000000
 .72     .3333576     .7199999    .0000001
 .76     .4300593     .7600000   -.0000000
 .80     .5392781     .8000000   -.0000000
 .84     .6568187     .8400000    .0000000
 .88     .7754564     .8800000   -.0000000
 .92     .8840248     .9200000    .0000000
 .96     .9663480     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000

A =    7.0 AND B =    2.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000002     .0800000   -.0000000
 .12     .0000026     .1200000   -.0000000
 .16     .0000185     .1600000   -.0000000
 .20     .0000845     .2000000   -.0000000
 .24     .0002899     .2400000   -.0000000
 .28     .0008150     .2800000    .0000000
 .32     .0019791     .3200000    .0000000
 .36     .0042944     .3600000   -.0000000
 .40     .0085197     .4000000   -.0000000
 .44     .0157085     .4400000    .0000000
 .48     .0272400     .4800000   -.0000000
 .52     .0448239     .5200000   -.0000000
 .56     .0704655     .5600000    .0000000
 .60     .1063757     .6000000   -.0000000
 .64     .1548112     .6400001   -.0000001
 .68     .2178248     .6800000    .0000000
 .72     .2969061     .7200000   -.0000000
 .76     .3924912     .7600000   -.0000000
 .80     .5033165     .8000000   -.0000000
 .84     .6255915     .8400000    .0000000
 .88     .7519631     .8800000   -.0000000
 .92     .8702407     .9199999    .0000001
 .96     .9618528     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    7.5 AND B =    2.0
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000009     .1200000   -.0000000
 .16    .0000078     .1600000    .0000000
 .20    .0000401     .2000000   -.0000000
 .24    .0001505     .2400000   -.0000000
 .28    .0004569     .2800000    .0000000
 .32    .0011856     .3200000    .0000000
 .36    .0027271     .3600000   -.0000000
 .40    .0056992     .4000000   -.0000000
 .44    .0110128     .4400000    .0000000
 .48    .0199299     .4800000   -.0000000
 .52    .0341022     .5200000   -.0000000
 .56    .0555749     .5599999    .0000001
 .60    .0867350     .6000000   -.0000000
 .64    .1301822     .6400000    .0000000
 .68    .1884932     .6799999    .0000001
 .72    .2638490     .7200000   -.0000000
 .76    .3574868     .7600000    .0000000
 .80    .4689374     .8000000   -.0000000
 .84    .5950005     .8400000   -.0000000
 .88    .7284062     .8800000   -.0000000
 .92    .8561083     .9199999    .0000001
 .96    .9571447     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000

A =    8.0 AND B =    2.0
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000003     .1200000    .0000000
 .16    .0000033     .1600000    .0000000
 .20    .0000189     .2000000   -.0000000
 .24    .0000779     .2400000    .0000000
 .28    .0002554     .2800000    .0000000
 .32    .0007081     .3200000    .0000000
 .36    .0017265     .3600001   -.0000001
 .40    .0038011     .4000000   -.0000000
 .44    .0076984     .4399999    .0000001
 .48    .0145405     .4800000   -.0000000
 .52    .0258745     .5200000   -.0000000
 .56    .0437162     .5600000    .0000000
 .60    .0705439     .6000000   -.0000000
 .64    .1092123     .6400000    .0000000
 .68    .1627501     .6800000    .0000000
 .72    .2339941     .7200000   -.0000000
 .76    .3250062     .7600000    .0000000
 .80    .4362076     .8000000   -.0000000
 .84    .5651570     .8400000   -.0000000
 .88    .7048837     .8800000   -.0000000
 .92    .8416790     .9200000    .0000000
 .96    .9522342     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.5 AND B =    2.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000001     .1200000    .0000000
 .16    .0000014     .1600000   -.0000000
 .20    .0000089     .2000000   -.0000000
 .24    .0000402     .2400000    .0000000
 .28    .0001423     .2800000    .0000000
 .32    .0004217     .3200000    .0000000
 .36    .0010901     .3600000    .0000000
 .40    .0025284     .4000000   -.0000000
 .44    .0053675     .4400000    .0000000
 .48    .0105816     .4800000    .0000000
 .52    .0195836     .5200000   -.0000000
 .56    .0343065     .5600000    .0000000
 .60    .0572451     .6000000   -.0000000
 .64    .0914231     .6400000    .0000000
 .68    .1402390     .6800000    .0000000
 .72    .2071300     .7200000   -.0000000
 .76    .2949777     .7600000    .0000000
 .80    .4051620     .8000000   -.0000000
 .84    .5361495     .8400000   -.0000000
 .88    .6814815     .8800000    .0000000
 .92    .8270006     .9200000    .0000000
 .96    .9471315     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000

A =    9.0 AND B =    2.0
  X      BETA(X)        INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000000 NOT DEFINED NOT DEFINED
 .16    .0000006     .1600000   -.0000000
 .20    .0000042     .2000000   -.0000000
 .24    .0000207     .2400000    .0000000
 .28    .0000791     .2800000    .0000000
 .32    .0002505     .3200000    .0000000
 .36    .0006865     .3600000    .0000000
 .40    .0016777     .4000000   -.0000000
 .44    .0037335     .4400000    .0000000
 .48    .0076828     .4800000    .0000000
 .52    .0147891     .5200000   -.0000000
 .56    .0268642     .5600000    .0000000
 .60    .0463574     .6000000   -.0000000
 .64    .0763810     .6400000    .0000000
 .68    .1206179     .6800000    .0000000
 .72    .1830354     .7200000   -.0000000
 .76    .2673064     .7600000    .0000000
 .80    .3758096     .8000000   -.0000000
 .84    .5080464     .8400000   -.0000000
 .88    .6582750     .8800000   -.0000000
 .92    .8121175     .9200000    .0000000
 .96    .9418462     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     2.5 AND B =     2.5
  X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0016645        .0400000    -.0000000
 .08     .0090042        .0800000    -.0000000
 .12     .0236974        .1200000    -.0000000
 .16     .0463959        .1600000     .0000000
 .20     .0771887        .2000000    -.0000000
 .24     .1157809        .2400000    -.0000000
 .28     .1615941        .2800000     .0000000
 .32     .2138329        .3200000     .0000000
 .36     .2715348        .3600000    -.0000000
 .40     .3336096        .4000000    -.0000000
 .44     .3988727        .4400000     .0000000
 .48     .4660741        .4800000     .0000000
 .52     .5339259        .5200000     .0000000
 .56     .6011273        .5600000     .0000000
 .60     .6663904        .6000000     .0000000
 .64     .7284652        .6400000     .0000000
 .68     .7861671        .6800000     .0000000
 .72     .8384059        .7200000     .0000000
 .76     .8842191        .7600000     .0000000
 .80     .9228113        .8000000     .0000000
 .84     .9536041        .8400000     .0000000
 .88     .9763026        .8800000     .0000000
 .92     .9909958        .9200000     .0000000
 .96     .9983355        .9600000     .0000000
1.00    1.0000000       1.0000000    0.0000000

A =     3.0 AND B =     2.5
  X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0004013        .0400000    -.0000000
 .08     .0030625        .0800000    -.0000000
 .12     .0098465        .1200001    -.0000001
 .16     .0222001        .1600000     .0000000
 .20     .0411741        .2000000    -.0000000
 .24     .0674439        .2400001    -.0000001
 .28     .1013313        .2800000     .0000000
 .32     .1428268        .3199999     .0000001
 .36     .1916134        .3600000    -.0000000
 .40     .2470920        .4000000    -.0000000
 .44     .3084076        .4400000    -.0000000
 .48     .3744778        .4800000    -.0000000
 .52     .4440233        .5200000     .0000000
 .56     .5156008        .5600000     .0000000
 .60     .5876390        .6000000     .0000000
 .64     .6584781        .6400000     .0000000
 .68     .7264146        .6800000     .0000000
 .72     .7897521        .7200000     .0000000
 .76     .8468603        .7600000     .0000000
 .80     .8962464        .8000000     .0000000
 .84     .9366451        .8400000     .0000000
 .88     .9671370        .8800000     .0000000
 .92     .9873232        .9200000     .0000000
 .95     .9976218        .9600000     .0000000
1.00    1.0000000       1.0000000    0.0000000
```

```
  INCOMPLETE BETA FUNCTION AND INVERSE

  A =    3.5 AND B =    2.5
    X      BETA(X)       INVERSE XC  ERROR TERM
  .04     .0000947     .0400000   -.0000000
  .08     .0010207     .0800000   -.0000000
  .12     .0040114     .1200000    .0000000
  .16     .0104212     .1600000   -.0000000
  .20     .0215599     .2000000   -.0000000
  .24     .0385909     .2400000    .0000000
  .28     .0624595     .2800000    .0000000
  .32     .0938425     .3200000   -.0000000
  .36     .1331127     .3600000    .0000000
  .40     .1803149     .4000000   -.0000000
  .44     .2351530     .4400000   -.0000000
  .48     .2969870     .4800000   -.0000000
  .52     .3648390     .5200000    .0000000
  .56     .4374079     .5600000   -.0000000
  .60     .5130959     .6000000   -.0000000
  .64     .5900434     .6400001   -.0000001
  .68     .6661770     .6800000    .0000000
  .72     .7392714     .7200000    .0000000
  .76     .8070292     .7600000    .0000000
  .80     .8671827     .8000000    .0000000
  .84     .9176295     .8400000    .0000000
  .88     .9566165     .8800000    .0000000
  .92     .9830123     .9200000    .0000000
  .96     .9967658     .9599999    .0000001
 1.00    1.0000000    1.0000000   0.0000000

  A =    4.0 AND B =    2.5
    X      BETA(X)       INVERSE XC  ERROR TERM
  .04     .0000220     .0400000   -.0000000
  .08     .0003347     .0800000   -.0000000
  .12     .0016085     .1200000    .0000000
  .16     .0048169     .1600000    .0000000
  .20     .0111213     .2000000   -.0000000
  .24     .0217628     .2400000    .0000000
  .28     .0379626     .2800000    .0000000
  .32     .0608310     .3200000   -.0000000
  .36     .0912844     .3600000    .0000000
  .40     .1299730     .4000000   -.0000000
  .44     .1772186     .4400000   -.0000000
  .48     .2329632     .4800000    .0000000
  .52     .2967302     .5200000    .0000000
  .56     .3675997     .5600000   -.0000000
  .60     .4441981     .6000000   -.0000000
  .64     .5247060     .6400000    .0000000
  .68     .6068862     .6800000   -.0000000
  .72     .6881361     .7200000    .0000000
  .76     .7655701     .7600000    .0000000
  .80     .8361409     .8000000    .0000000
  .84     .8968154     .8400000    .0000000
  .88     .9448285     .8800000    .0000000
  .92     .9780729     .9200000    .0000000
  .96     .9957638     .9599999    .0000001
 1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    4.5 AND B =    2.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000050      .0400001   -.0000001
 .08     .0001083      .0800000   -.0000000
 .12     .0006366      .1200000    .0000000
 .16     .0021984      .1600000    .0000000
 .20     .0056660      .2000000   -.0000000
 .24     .0121258      .2400000    .0000000
 .28     .0228056      .2800000   -.0000000
 .32     .0389898      .3200000   -.0000000
 .36     .0619243      .3600000    .0000000
 .40     .0927180      .4000000   -.0000000
 .44     .1322436      .4400000   -.0000000
 .48     .1810417      .4800000    .0000000
 .52     .2392315      .5200000    .0000000
 .56     .3064323      .5600000   -.0000000
 .60     .3817005      .6000000   -.0000000
 .64     .4634862      .6400000    .0000000
 .68     .5496150      .6800000   -.0000000
 .72     .6373044      .7200000    .0000000
 .76     .7232229      .7600000    .0000000
 .80     .8036071      .8000000    .0000000
 .84     .8744599      .8400000    .0000000
 .88     .9318683      .8800000    .0000000
 .92     .9725197      .9200000    .0000000
 .96     .9946130      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000

A =    5.0 AND B =    2.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000011      .0400000   -.0000000
 .08     .0000347      .0800000   -.0000000
 .12     .0002493      .1200000    .0000000
 .16     .0009927      .1600000   -.0000000
 .20     .0028568      .2000000   -.0000000
 .24     .0066880      .2400000    .0000000
 .28     .0135657      .2800000   -.0000000
 .32     .0247529      .3200000   -.0000000
 .36     .0416215      .3599999    .0000001
 .40     .0655576      .4000000   -.0000000
 .44     .0978493      .4400000   -.0000000
 .48     .1395635      .4800000    .0000000
 .52     .1914156      .5200000   -.0000000
 .56     .2536388      .5600000   -.0000000
 .60     .3258593      .6000000   -.0000000
 .64     .4069866      .6400000   -.0000000
 .68     .4951272      .6800000   -.0000000
 .72     .5875363      .7200000    .0000000
 .76     .6806218      .7600000    .0000000
 .80     .7700249      .8000000    .0000000
 .84     .8508121      .8400000    .0000000
 .88     .9178352      .8800000    .0000000
 .92     .9663712      .9200000    .0000000
 .96     .9933113      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000
```

```
 INCOMPLETE BETA FUNCTION AND INVERSE

 A =    5.5 AND B =    2.5
   X      BETA(X)      INVERSE XC  ERROR TERM
 .04    .0000003    .0400000   -.0000000
 .08    .0000110    .0800000   -.0000000
 .12    .0000967    .1200000    .0000000
 .16    .0004442    .1600000    .0000000
 .20    .0014277    .2000000   -.0000000
 .24    .0036569    .2400000    .0000000
 .28    .0080015    .2800000    .0000000
 .32    .0155860    .3200000   -.0000000
 .36    .0277539    .3600000    .0000000
 .40    .0459996    .4000000   -.0000000
 .44    .0718701    .4400000    .0000000
 .48    .1068366    .4799999    .0000001
 .52    .1521436    .5200000   -.0000000
 .56    .2086371    .5600000    .0000000
 .60    .2765842    .6000000   -.0000000
 .64    .3554907    .6400000   -.0000000
 .68    .4439321    .6800000   -.0000000
 .72    .5394162    .7200000    .0000000
 .76    .6382993    .7600000   -.0000000
 .80    .7357931    .8000000    .0000000
 .84    .8261099    .8400000    .0000000
 .88    .9028305    .8800000    .0000000
 .92    .9596488    .9200000    .0000000
 .96    .9918574    .9600000    .0000000
1.00   1.0000000   1.0000000   0.0000000

 A =    6.0 AND B =    2.5
   X      BETA(X)      INVERSE XC  ERROR TERM
 .04    .0000001 NOT DEFINED NOT DEFINED
 .08    .0000035    .0800000   -.0000000
 .12    .0000372    .1200000    .0000000
 .16    .0001972    .1600000    .0000000
 .20    .0007080    .2000000   -.0000000
 .24    .0019846    .2400000    .0000000
 .28    .0046852    .2800000    .0000000
 .32    .0097444    .3200000   -.0000000
 .36    .0183793    .3600000    .0000000
 .40    .0320615    .4000000   -.0000000
 .44    .0524500    .4400000    .0000000
 .48    .0812821    .4800000    .0000000
 .52    .1202230    .5200000   -.0000000
 .56    .1706753    .5600000    .0000000
 .60    .2335551    .6000000   -.0000000
 .64    .3090440    .6400000   -.0000000
 .68    .3963322    .6800001   -.0000001
 .72    .4933749    .7200000    .0000000
 .76    .5966929    .7600000    .0000000
 .80    .7012642    .8000000    .0000000
 .84    .8005765    .8400000    .0000000
 .88    .8869549    .8800000    .0000000
 .92    .9523760    .9200000    .0000000
 .96    .9902506    .9600000    .0000000
1.00   1.0000000   1.0000000   0.0000000
```

```
 INCOMPLETE BETA FUNCTION AND INVERSE

 A =   6.5 AND B =   2.5
   X     BETA(X)     INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000011    .0800000   -.0000000
 .12    .0000142    .1199999    .0000001
 .16    .0000870    .1600000    .0000000
 .20    .0003488    .2000000   -.0000000
 .24    .0010701    .2400000    .0000000
 .28    .0027259    .2800000   -.0000000
 .32    .0060543    .3200001   -.0000001
 .36    .0120976    .3600000    .0000000
 .40    .0222157    .4000000   -.0000000
 .44    .0380609    .4400000    .0000000
 .48    .0615041    .4800000    .0000000
 .52    .0945073    .5200000   -.0000000
 .56    .1389358    .5600000    .0000000
 .60    .1963136    .6000000   -.0000000
 .64    .2675234    .6400000   -.0000000
 .68    .3524684    .6800000    .0000000
 .72    .4497149    .7200000    .0000000
 .76    .5561549    .7600000   -.0000000
 .80    .6667461    .8000000    .0000000
 .84    .7744194    .8399999    .0000001
 .88    .8703081    .8800000    .0000000
 .92    .9445781    .9200000    .0000000
 .96    .9884907    .9600000    .0000000
1.00   1.0000000   1.0000000   0.0000000

 A =   7.0 AND B =   2.5
   X     BETA(X)     INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000003    .0800000   -.0000000
 .12    .0000054    .1200000   -.0000000
 .16    .0000381    .1600000    .0000000
 .20    .0001708    .2000000   -.0000000
 .24    .0005736    .2400000   -.0000000
 .28    .0015770    .2800000    .0000000
 .32    .0037409    .3200000    .0000000
 .36    .0079203    .3600000   -.0000000
 .40    .0153135    .4000000   -.0000000
 .44    .0274804    .4400000    .0000000
 .48    .0463133    .4800000    .0000000
 .52    .0739478    .5200000   -.0000000
 .56    .1126008    .5600000    .0000000
 .60    .1643269    .6000000   -.0000000
 .64    .2306899    .6400000   -.0000000
 .68    .3123565    .6800000    .0000000
 .72    .4086297    .7200000    .0000000
 .76    .5169605    .7600000   -.0000000
 .80    .6325035    .8000000    .0000000
 .84    .7478291    .8400000    .0000000
 .88    .8529866    .8800000    .0000000
 .92    .9362815    .9200000    .0000000
 .96    .9865778    .9600000    .0000000
1.00   1.0000000   1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    7.5 AND B =    2.5
  X      BETA(X)        INVERSE XC   ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000001      .0800001    -.0000001
 .12    .0000020      .1200000    -.0000000
 .16    .0000166      .1600000    -.0000000
 .20    .0000832      .2000000    -.0000000
 .24    .0003060      .2400000    -.0000000
 .28    .0009078      .2800000    -.0000000
 .32    .0023002      .3200000     .0000000
 .36    .0051606      .3600000    -.0000000
 .40    .0105068      .4000000    -.0000000
 .44    .0197519      .4400000     .0000000
 .48    .0347230      .4800000    -.0000000
 .52    .0576201      .5200000    -.0000000
 .56    .0908956      .5600000     .0000000
 .60    .1370368      .6000000    -.0000000
 .64    .1982323      .6400000    -.0000000
 .68    .2759204      .6800000     .0000000
 .72    .3702257      .7199999     .0000001
 .76    .4793184      .7600000    -.0000000
 .80    .5987614      .8000000    -.0000000
 .84    .7209793      .8400000     .0000000
 .88    .8350838      .8800000     .0000000
 .92    .9275134      .9200000     .0000000
 .96    .9845127      .9600000     .0000000
1.00   1.0000000     1.0000000    0.0000000

A =    8.0 AND B =    2.5
  X      BETA(X)        INVERSE XC   ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000008      .1200001    -.0000001
 .16    .0000072      .1600000     .0000000
 .20    .0000404      .2000000    -.0000000
 .24    .0001624      .2400001    -.0000001
 .28    .0005203      .2800000    -.0000000
 .32    .0014082      .3200000     .0000000
 .36    .0033482      .3600000    -.0000000
 .40    .0071788      .4000000    -.0000000
 .44    .0141395      .4400000     .0000000
 .48    .0259314      .4800000    -.0000000
 .52    .0447283      .5200000    -.0000000
 .56    .0731102      .5599999     .0000001
 .60    .1138893      .6000000    -.0000000
 .64    .1697976      .6400001    -.0000001
 .68    .2430165      .6800000     .0000000
 .72    .3345381      .7199999     .0000001
 .76    .4433788      .7600000    -.0000000
 .80    .5657074      .8000000    -.0000000
 .84    .6940267      .8400000     .0000000
 .88    .8166899      .8800000     .0000000
 .92    .9133016      .9200000     .0000000
 .96    .9822963      .9600000     .0000000
1.00   1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.5 AND B =    2.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000003     .1200000    .0000000
 .16     .0000031     .1600000    .0000000
 .20     .0000195     .2000000   -.0000000
 .24     .0000859     .2400000    .0000000
 .28     .0002970     .2800000    .0000000
 .32     .0008587     .3200000    .0000000
 .36     .0021639     .3600000    .0000000
 .40     .0048864     .4000000   -.0000000
 .44     .0100848     .4399999    .0000001
 .48     .0192971     .4800000   -.0000000
 .52     .0346024     .5200000   -.0000000
 .56     .0586125     .5600000    .0000000
 .60     .0943575     .6000000   -.0000000
 .64     .1450168     .6400000    .0000000
 .68     .2134571     .6799999    .0000001
 .72     .3015471     .7200000   -.0000000
 .76     .4092434     .7600001   -.0000001
 .80     .5334960     .8000000   -.0000000
 .84     .6671118     .8400000   -.0000000
 .88     .7978870     .8800000   -.0000000
 .92     .9086741     .9200000    .0000000
 .96     .9799300     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000

A =    9.0 AND B =    2.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000001     .1200000    .0000000
 .16     .0000013     .1600000    .0000000
 .20     .0000094     .2000000   -.0000000
 .24     .0000453     .2400000    .0000000
 .28     .0001689     .2800000   -.0000000
 .32     .0005218     .3200000    .0000000
 .36     .0013936     .3600000    .0000000
 .40     .0033147     .4000000   -.0000000
 .44     .0071689     .4400000    .0000000
 .48     .0143138     .4800000    .0000000
 .52     .0266852     .5200000   -.0000000
 .56     .0468489     .5600000    .0000000
 .60     .0779524     .6000000   -.0000000
 .64     .1235195     .6400000    .0000000
 .68     .1870245     .6800000    .0000000
 .72     .2711898     .7200000   -.0000000
 .76     .3769714     .7600000    .0000000
 .80     .5022511     .8000000   -.0000000
 .84     .6403589     .8400000    .0000000
 .88     .7787579     .8800000   -.0000000
 .92     .8986587     .9200000    .0000000
 .96     .9774154     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     3.0 AND B =     3.0
   X        BETA(X)        INVERSE XC   ERROR TERM
 .04      .0006022      .0400000    -.0000000
 .08      .0045253      .0800000    -.0000000
 .12      .0143189      .1200001    -.0000001
 .16      .0317587      .1600000     .0000000
 .20      .0579200      .2000000    -.0000000
 .24      .0932512      .2400000    -.0000000
 .28      .1376478      .2800000     .0000000
 .32      .1905263      .3200000     .0000000
 .36      .2508973      .3600000    -.0000000
 .40      .3174400      .4000000    -.0000000
 .44      .3885753      .4400000     .0000000
 .48      .4625400      .4799999     .0000001
 .52      .5374600      .5199999     .0000001
 .56      .6114247      .5600000     .0000000
 .60      .6825600      .6000000     .0000000
 .64      .7491027      .6400000     .0000000
 .68      .8094737      .6800000     .0000000
 .72      .8623522      .7199999     .0000001
 .76      .9067488      .7599999     .0000001
 .80      .9420800      .8000000     .0000000
 .84      .9682413      .8400000     .0000000
 .88      .9856811      .8800000     .0000000
 .92      .9954747      .9200000     .0000000
 .96      .9993978      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000

A =     3.5 AND B =     3.0
   X        BETA(X)        INVERSE XC   ERROR TERM
 .04      .0001487      .0400000    -.0000000
 .08      .0015764      .0800000    -.0000000
 .12      .0060928      .1200000     .0000000
 .16      .0155592      .1600000    -.0000000
 .20      .0316269      .2000000    -.0000000
 .24      .0555915      .2400000     .0000000
 .28      .0883088      .2799999     .0000001
 .32      .1301513      .3200000    -.0000000
 .36      .1809954      .3600001    -.0000001
 .40      .2402319      .4000000    -.0000000
 .44      .3067978      .4400000    -.0000000
 .48      .3792249      .4800000    -.0000000
 .52      .4557046      .5200000     .0000000
 .56      .5341656      .5600000    -.0000000
 .60      .6123651      .6000000     .0000000
 .64      .6879917      .6400000     .0000000
 .68      .7587733      .6800000     .0000000
 .72      .8226263      .7200000     .0000000
 .76      .8777387      .7600000     .0000000
 .80      .9227626      .8000000     .0000000
 .84      .9569402      .8399999     .0000001
 .88      .9802680      .8800000     .0000000
 .92      .9936642      .9200000     .0000000
 .96      .9991436      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    4.0 AND B =    3.0
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000360      .0400000    -.0000000
 .08     .0005384      .0800000    -.0000000
 .12     .0025431      .1200000     .0000000
 .16     .0074816      .1600000    -.0000000
 .20     .0169600      .2000000    -.0000000
 .24     .0325671      .2400000     .0000000
 .28     .0557124      .2800000     .0000000
 .32     .0874932      .3200000    -.0000000
 .36     .1285914      .3600000     .0000000
 .40     .1792000      .4000000    -.0000000
 .44     .2389786      .4400000    -.0000000
 .48     .3070388      .4800000    -.0000000
 .52     .3819588      .5200000     .0000000
 .56     .4618279      .5600000    -.0000000
 .60     .5443200      .6000000    -.0000000
 .64     .6267368      .6400000     .0000000
 .68     .7064407      .6800000     .0000000
 .72     .7804168      .7200000     .0000000
 .76     .8460648      .7600000     .0000000
 .80     .9011200      .8000[illegible]0     .0000000
 .84     .9439641      .8399[illegible]99     .0000001
 .88     .9739053      .8800000     .0000000
 .92     .9914879      .9200000     .0000000
 .96     .9988316      .9600000     .0000000
1.00    1.0000000     1.0000000     0.0000000

A =    4.5 ANC B =    3.0
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000086      .0400001    -.0000001
 .08     .0001809      .0800000    -.0000000
 .12     .0010447      .1200000     .0000000
 .16     .0035420      .1600000     .0000000
 .20     .0089586      .2000000    -.0000000
 .24     .0188019      .2400000     .0000000
 .28     .0346555      .2799999     .0000001
 .32     .0580245      .3200000    -.0000000
 .36     .0901833      .3600000     .0000000
 .40     .1320365      .4000000    -.0000000
 .44     .1839991      .4400000    -.0000000
 .48     .2459036      .4800000     .0000000
 .52     .3169394      .5200000     .0000000
 .56     .3956300      .5600000    -.0000000
 .60     .4798533      .6000000    -.0000000
 .64     .5669088      .6400000    -.0000000
 .68     .6536362      .6800000     .0000000
 .72     .7365897      .7200000     .0000000
 .76     .8122711      .7600000     .0000000
 .80     .8774259      .8000000     .0000000
 .84     .9294054      .8400000     .0000000
 .88     .9665577      .8799999     .0000001
 .92     .9889319      .9200000     .0000000
 .96     .9984571      .9600000     .0000000
1.00    1.0000000     1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

  A =    5.0 AND B =    3.0
    X       BETA(X)        INVERSE XC  ERROR TERM
  .04      .0000020      .0400000    -.0000000
  .08      .0000600      .0800000    -.0000000
  .12      .0004234      .1200000     .0000000
  .16      .0016551      .1600000    -.0000000
  .20      .0046720      .2000000    -.0000000
  .24      .0107209      .2399999     .0000001
  .28      .0212996      .2800000    -.0000000
  .32      .0380373      .3200000    -.0000000
  .36      .0625462      .3600000     .0000000
  .40      .0962560      .4000000    -.0000000
  .44      .1402448      .4400000    -.0000000
  .48      .1950779      .4800000     .0000000
  .52      .2606679      .5200000     .0000000
  .56      .3361667      .5600000    -.0000000
  .60      .4199040      .6000000    -.0000000
  .64      .5093831      .6400000     .0000000
  .68      .6013469      .6800000    -.0000000
  .72      .6919265      .7200000     .0000000
  .76      .7768850      .7600000     .0000000
  .80      .8519680      .8000000     .0000000
  .84      .9133745      .8400000     .0000000
  .88      .9583612      .8800000     .0000000
  .92      .9859860      .9199999     .0000001
  .96      .9980162      .9600000     .0000000
 1.00     1.0000000     1.0000000    0.0000000

  A =    5.5 AND B =    3.0
    X       BETA(X)        INVERSE XC  ERROR TERM
  .04      .0000005      .0400000    -.0000000
  .08      .0000196      .0800000    -.0000000
  .12      .0001697      .1200000     .0000000
  .16      .0007647      .1599999     .0000001
  .20      .0024099      .2000000    -.0000000
  .24      .0060481      .2400000     .0000000
  .28      .0129558      .2800000     .0000000
  .32      .0246859      .3200000    -.0000000
  .36      .0429610      .3600000     .0000000
  .40      .0695236      .4000000    -.0000000
  .44      .1059537      .4400001    -.0000001
  .48      .1534675      .4800000     .0000000
  .52      .2127113      .5200000    -.0000000
  .56      .2835701      .5600000    -.0000000
  .60      .3650098      .6000000    -.0000000
  .64      .4549745      .6399999     .0000001
  .68      .5503634      .6800000    -.0000000
  .72      .6471117      .7200000     .0000000
  .76      .7404022      .7600000     .0000000
  .80      .8250368      .8000000     .0000000
  .84      .8959964      .8400000     .0000000
  .88      .9492212      .8800000     .0000000
  .92      .9826432      .9199999     .0000001
  .96      .9975051      .9600000     .0000000
 1.00     1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     6.0 AND B =     3.0
   X       BETA(X)         INVERSE XC  ERROR TERM
 .04      .0000001       .0400001    -.0000001
 .08      .0000064       .0800000    -.0000000
 .12      .0000673       .1200000     .0000000
 .16      .0003499       .1600000    -.0000000
 .20      .0012314       .2000000    -.0000000
 .24      .0033805       .2400000     .0000000
 .28      .0078097       .2799999     .0000001
 .32      .0158811       .3200000    -.0000000
 .36      .0292594       .3600000     .0000000
 .40      .0498074       .4000000    -.0000000
 .44      .0794247       .4400000     .0000000
 .48      .1198402       .4800000     .0000000
 .52      .1723681       .5200000    -.0000000
 .56      .2376483       .5600000     .0000000
 .60      .3153946       .6000000    -.0000000
 .64      .4041805       .6400000    -.0000000
 .68      .5012977       .6800000    -.0000000
 .72      .6027284       .7200000     .0000000
 .76      .7032777       .7600000     .0000000
 .80      .7969178       .8000000     .0000000
 .84      .8774020       .8400000     .0000000
 .88      .9392108       .8800000     .0000000
 .92      .9788995       .9199999     .0000001
 .96      .9969203       .9600000     .0000000
1.00     1.0000000      1.0000000    0.0000000

A =     6.5 AND B =     3.0
   X       BETA(X)         INVERSE XC  ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000020       .0800000    -.0000000
 .12      .0000265       .1199999     .0000001
 .16      .0001588       .1600000    -.0000000
 .20      .0006240       .2000000    -.0000000
 .24      .0018742       .2400000     .0000000
 .28      .0046704       .2800000     .0000000
 .32      .0101381       .3200000    -.0000000
 .36      .0197791       .3600000     .0000000
 .40      .0354256       .4000000    -.0000000
 .44      .0591265       .4400000     .0000000
 .48      .0929639       .4800000     .0000000
 .52      .1398041       .5200000    -.0000000
 .56      .1979971       .5600000     .0000000
 .60      .2710469       .6000000    -.0000000
 .64      .3572963       .6400000    -.0000000
 .68      .4546013       .6800000    -.0000000
 .72      .5592605       .7200000     .0000000
 .76      .6659199       .7600000     .0000000
 .80      .7678851       .8000000     .0000000
 .84      .8577280       .8400000     .0000000
 .88      .9283694       .8800000     .0000000
 .92      .9747537       .9200000     .0000000
 .96      .9962588       .9600000     .0000000
1.00     1.0000000      1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     7.0 AND B =     3.0
  X       BETA(X)          INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000007      .0800000     -.0000000
 .12     .0000103      .1200000     -.0000000
 .16     .0000715      .1600000      .0000000
 .20     .0003139      .2000000     -.0000000
 .24     .0010316      .2400000     -.0000000
 .28     .0027735      .2800000      .0000000
 .32     .0064277      .3200001     -.0000001
 .36     .0132818      .3600000      .0000000
 .40     .0250348      .4000000     -.0000000
 .44     .0437436      .4400000      .0000000
 .48     .0716881      .4800000      .0000000
 .52     .1111469      .5200000     -.0000000
 .56     .1640878      .5600000      .0000000
 .60     .2317870      .6000000     -.0000000
 .64     .3144075      .6400000     -.0000000
 .68     .4105864      .6800001     -.0000001
 .72     .5170982      .7200000      .0000000
 .76     .6286889      .7600000     -.0000000
 .80     .7381975      .8000000      .0000000
 .84     .8371123      .8400000      .0000000
 .88     .9167411      .8800000      .0000000
 .92     .9702068      .9200000      .0000000
 .96     .9955176      .9600000      .0000000
1.00    1.0000000     1.0000000     0.0000000

A =     7.5 ANC B =     3.0
  X       BETA(X)          INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000002      .0800001     -.0000001
 .12     .0000040      .1200000     -.0000000
 .16     .0000320      .1600000      .0000000
 .20     .0001568      .2000000     -.0000000
 .24     .0005642      .2400000     -.0000000
 .28     .0016367      .2800000      .0000000
 .32     .0040504      .3200000      .0000000
 .36     .0088658      .3600000     -.0000000
 .40     .0175898      .4000000     -.0000000
 .44     .0321828      .4400000      .0000000
 .48     .0549862      .4800000      .0000000
 .52     .0885472      .5200000     -.0000000
 .56     .1353313      .5600000      .0000000
 .60     .1973221      .6000000     -.0000000
 .64     .2755288      .6400000     -.0000000
 .68     .3694467      .6800000      .0000000
 .72     .4765453      .7200000      .0000000
 .76     .5918960      .7600000      .0000000
 .80     .7080955      .8000000      .0000000
 .84     .8150916      .8400000      .0000000
 .88     .9043738      .8800000      .0000000
 .92     .9652621      .9200000      .0000000
 .96     .9946942      .9600000      .0000000
1.00    1.0000000     1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.0 AND B =    3.0
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000001 NOT DEFINED NOT DEFINED
 .12    .0000015    .1200001   -.0000001
 .16    .0000142    .1600000    .0000000
 .20    .0000779    .2000000   -.0000000
 .24    .0003069    .2400000   -.0000000
 .28    .0009605    .2799999    .0000001
 .32    .0025384    .3200000    .0000000
 .36    .0058864    .3600000   -.0000000
 .40    .0122946    .4000000   -.0000000
 .44    .0235583    .4400000    .0000000
 .48    .0419713    .4800000   -.0000000
 .52    .0702161    .5200000   -.0000000
 .56    .1111243    .5600000    .0000000
 .60    .1672898    .6000000   -.0000000
 .64    .2405373    .6400000   -.0000000
 .68    .3312788    .6800000    .0000000
 .72    .4378290    .7200000    .0000000
 .76    .5558051    .7600000   -.0000000
 .80    .6777995    .8000000    .0000000
 .84    .7935995    .8400000    .0000000
 .88    .8913182    .8800000    .0000000
 .92    .9599246    .9200000    .0000000
 .96    .9937863    .9600000    .0000000
1.00   1.0000000   1.0000000   0.0000000

A =    8.5 AND B =    3.0
  X      BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000006    .1200000    .0000000
 .16    .0000063    .1600000   -.0000000
 .20    .0000385    .2000000   -.0000000
 .24    .0001660    .2400000    .0000000
 .28    .0005608    .2800000    .0000000
 .32    .0015829    .3200000    .0000000
 .36    .0038893    .3600000   -.0000000
 .40    .0085529    .4000000   -.0000000
 .44    .0171662    .4400000    .0000000
 .48    .0318958    .4800000   -.0000000
 .52    .0554447    .5200000   -.0000000
 .56    .0908804    .5599999    .0000001
 .60    .1412913    .6000000   -.0000000
 .64    .2032507    .6400000   -.0000000
 .68    .2961002    .6800000    .0000000
 .72    .4011090    .7200000    .0000000
 .76    .5206356    .7600000   -.0000000
 .80    .6475088    .8000000    .0000000
 .84    .7709649    .8400000    .0000000
 .88    .8776267    .8800000    .0000000
 .92    .9542011    .9200000    .0000000
 .96    .9927917    .9600000    .0000000
1.00   1.0000000   1.0000000   0.0000000
```

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# INCOMPLETE BETA FUNCTION AND INVERSE

```
 A =     9.0 AND B =     3.0
   X       BETA(X)        INVERSE  XC   ERROR TERM
 .04      .0000000  NOT  DEFINED  NOT  DEFINED
 .08      .0000000  NOT  DEFINED  NOT  DEFINED
 .12      .0000002       .1200000       .0000000
 .16      .0000028       .1600000       .0000000
 .20      .0000189       .2000000      -.0000000
 .24      .0000894       .2400000       .0000000
 .28      .0003259       .2800000       .0000000
 .32      .0009826       .3200000       .0000000
 .36      .0025585       .3600000       .0000000
 .40      .0059245       .4000000      -.0000000
 .44      .0124564       .4399999       .0000001
 .48      .0241413       .4800000      -.0000000
 .52      .0436112       .5200000      -.0000000
 .56      .0740499       .5600000       .0000000
 .60      .1189168       .6000000      -.0000000
 .64      .1814410       .6400001      -.0000001
 .68      .2638673       .6800000       .0000000
 .72      .3664868       .7199999       .0000001
 .76      .4865654       .7600000      -.0000000
 .80      .6174015       .8000000      -.0000000
 .84      .7479110       .8400000       .0000000
 .88      .8633530       .8799999       .0000001
 .92      .9481000       .9200000       .0000000
 .96      .9917087       .9600000       .0000000
1.00     1.0000000      1.0000000      0.0000000

 A =     3.5 AND B =     3.5
   X       BETA(X)        INVERSE  XC   ERROR TERM
 .04      .0002203       .0400000      -.0000000
 .08      .0022981       .0800000      -.0000000
 .12      .0087361       .1200000       .0000000
 .16      .0219331       .1599999       .0000001
 .20      .0438115       .2000000      -.0000000
 .24      .0756422       .2400000       .0000000
 .28      .1179749       .2800000       .0000000
 .32      .1706364       .3199999       .0000001
 .36      .2327766       .3600000      -.0000000
 .40      .3029507       .4000000      -.0000000
 .44      .3792263       .4400000       .0000000
 .48      .4593106       .4800000      -.0000000
 .52      .5406894       .5199999       .0000001
 .56      .6207737       .5600000       .0000000
 .60      .6970493       .6000000       .0000000
 .64      .7672234       .6400000       .0000000
 .68      .8293636       .6800000       .0000000
 .72      .8820251       .7200000       .0000000
 .76      .9243578       .7600000       .0000000
 .80      .9551885       .8000000       .0000000
 .84      .9780669       .8400000       .0000000
 .88      .9912639       .8800000       .0000000
 .92      .9977019       .9200000       .0000000
 .96      .9997797       .9600000       .0000000
1.00     1.0000000      1.0000000      0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    4.0 AND B =    3.5
  X       BETA(X)       INVERSE XC   ERROR TERM
 .04    .0000554     .0400000    -.0000000
 .08    .0008148     .0800000    -.0000000
 .12    .0037833     .1200000     .0000000
 .16    .0109358     .1600000    -.0000000
 .20    .0243445     .2000000    -.0000000
 .24    .0458824     .2400000     .0000000
 .28    .0769977     .2800000     .0000000
 .32    .1185560     .3200000    -.0000000
 .36    .1707450     .3600000     .0000000
 .40    .2330378     .4000000    -.0000000
 .44    .3042096     .4400000    -.0000000
 .48    .3824026     .4800000    -.0000000
 .52    .4652335     .5200000     .0000000
 .56    .5499371     .5600000    -.0000000
 .60    .6335401     .6000000     .0000000
 .64    .7130571     .6399999     .0000001
 .68    .7857007     .6800000     .0000000
 .72    .8490958     .7199999     .0000001
 .76    .9014873     .7600000     .0000000
 .80    .9419266     .8000000     .0000000
 .84    .9704207     .8400000     .0000000
 .88    .9880178     .8800000     .0000000
 .92    .9967956     .9200000     .0000000
 .96    .9996878     .9600000     .0000000
1.00   1.0000000    1.0000000    0.0000000

A =    4.5 AND B =    3.5
  X       BETA(X)       INVERSE XC   ERROR TERM
 .04    .0000137     .0400000    -.0000000
 .08    .0002835     .0800000    -.0000000
 .12    .0016086     .1200000     .0000000
 .16    .0053559     .1600000     .0000000
 .20    .0132951     .2000000    -.0000000
 .24    .0273697     .2400000     .0000000
 .28    .0494530     .2800000     .0000000
 .32    .0811167     .3200000    -.0000000
 .36    .1234311     .3600000     .0000000
 .40    .1768110     .4000000    -.0000000
 .44    .2409160     .4400000    -.0000000
 .48    .3146107     .4800001    -.0000001
 .52    .3959896     .5200000     .0000000
 .56    .4824635     .5599999     .0000001
 .60    .5709099     .6000000     .0000000
 .64    .6578780     .6400000     .0000000
 .68    .7398441     .6800000     .0000000
 .72    .8135034     .7200000     .0000000
 .76    .8760855     .7600000     .0000000
 .80    .9256723     .8000000     .0000000
 .84    .9614898     .8400000     .0000000
 .88    .9841365     .8800000     .0000000
 .92    .9956874     .9200000     .0000000
 .96    .9995730     .9600000     .0000000
1.00   1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    5.0 AND B =    3.5
  X      BETA(X)       INVERSE XC   ERROR TERM
 .04     .0000033     .0400000   -.0000000
 .08     .0000971     .0800000   -.0000000
 .12     .0006733     .1199999    .0000001
 .16     .0025836     .1599999    .0000001
 .20     .0071543     .2000000   -.0000000
 .24     .0160946     .2400000    .0000000
 .28     .0313267     .2800000   -.0000000
 .32     .0547699     .3200000   -.0000000
 .36     .0881060     .3600000    .0000000
 .40     .1325496     .4000000   -.0000000
 .44     .1886478     .4400000   -.0000000
 .48     .2561263     .4800000    .0000000
 .52     .3338009     .5200000    .0000000
 .56     .4195659     .5600000   -.0000000
 .60     .5104678     .6000000   -.0000000
 .64     .6028717     .6400000    .0000000
 .68     .6927172     .6800000    .0000000
 .72     .7758592     .7200000    .0000000
 .76     .8484796     .7600000    .0000000
 .80     .9075463     .8000000    .0000000
 .84     .9512833     .8400000    .0000000
 .88     .9795959     .8800000    .0000000
 .92     .9943616     .9200000    .0000000
 .96     .9994327     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000

A =    5.5 AND B =    3.5
  X      BETA(X)       INVERSE XC   ERROR TERM
 .04     .0000008     .0400000   -.0000000
 .08     .0000328     .0800000   -.0000000
 .12     .0002781     .1200000    .0000000
 .16     .0012301     .1600000    .0000000
 .20     .0038011     .2000000   -.0000000
 .24     .0093480     .2400000    .0000000
 .28     .0196079     .2800001   -.0000001
 .32     .0365558     .3200000   -.0000000
 .36     .0621977     .3600000    .0000000
 .40     .0983242     .4000000   -.0000000
 .44     .1462503     .4400000   -.0000000
 .48     .2065682     .4800000    .0000000
 .52     .2789436     .5200000    .0000000
 .56     .3619799     .5600000   -.0000000
 .60     .4531796     .6000000   -.0000000
 .64     .5490186     .6400000    .0000000
 .68     .6451523     .6800000    .0000000
 .72     .7367590     .7200000    .0000000
 .76     .8190168     .7600000    .0000000
 .80     .8876964     .8000000    .0000000
 .84     .9398290     .8400000    .0000000
 .88     .9743798     .8800000    .0000000
 .92     .9928043     .9200000    .0000000
 .96     .9992644     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.0 AND B =    3.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000002      .0400000    -.0000000
 .08     .0000109      .0800000    -.0000000
 .12     .0001135      .1200000     .0000000
 .16     .0005790      .1600000    -.0000000
 .20     .0019973      .2000000    -.0000000
 .24     .0053710      .2400000     .0000000
 .28     .0121448      .2800000     .0000000
 .32     .0241525      .3200000    -.0000000
 .36     .0434809      .3600000     .0000000
 .40     .0722568      .4000000    -.0000000
 .44     .1123770      .4400001    -.0000001
 .48     .1652073      .4800000     .0000000
 .52     .2312835      .5200000    -.0000000
 .56     .3100540      .5600000    -.0000000
 .60     .3997027      .6000000    -.0000000
 .64     .4970937      .6400000     .0000000
 .68     .5978740      .6800001    -.0000001
 .72     .6967636      .7200000     .0000000
 .76     .7880508      .7600000     .0000000
 .80     .8662899      .8000000     .0000000
 .84     .9271702      .8399999     .0000001
 .88     .9684790      .8800000     .0000000
 .92     .9910028      .9200000     .0000000
 .96     .9990655      .9600000     .0000000
1.00    1.0000000     1.0000000    0.0000000

A =    6.5 AND B =    3.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000036      .0800000    -.0000000
 .12     .0000459      .1200000     .0000000
 .16     .0002699      .1600000     .0000000
 .20     .0010393      .2000000    -.0000000
 .24     .0030568      .2400000     .0000000
 .28     .0074529      .2800000     .0000000
 .32     .0158148      .3200000    -.0000000
 .36     .0301336      .3600000     .0000000
 .40     .0526591      .4000000    -.0000000
 .44     .0856642      .4400000     .0000000
 .48     .1311349      .4800000     .0000000
 .52     .1904142      .5200000    -.0000000
 .56     .2638405      .5600000     .0000000
 .60     .3504332      .6000000    -.0000000
 .64     .4476803      .6400000    -.0000000
 .68     .5514935      .6800000    -.0000000
 .72     .6563866      .7200000     .0000000
 .76     .7559300      .7600000     .0000000
 .80     .8435064      .8000000     .0000000
 .84     .9133635      .8400000     .0000000
 .88     .9618912      .8800000     .0000000
 .92     .9889462      .9200000     .0000000
 .96     .9988334      .9600000     .0000000
1.00    1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     7.0 AND B =     3.5
  X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0000000 NOT  DEFINED NOT DEFINED
 .08     .0000012       .0800000   -.0000000
 .12     .0000184       .1200000   -.0000000
 .16     .0001247       .1600000   -.0000000
 .20     .0005361       .2000000   -.0000000
 .24     .0017250       .2400000   -.0000000
 .28     .0045360       .2800000   -.0000000
 .32     .0102727       .3200001   -.0000001
 .36     .0207221       .3600000    .0000000
 .40     .0380908       .4000000   -.0000000
 .44     .0648349       .4400000    .0000000
 .48     .1033824       .4800000    .0000000
 .52     .1557624       .5200000   -.0000000
 .56     .2231746       .5600000    .0000000
 .60     .3055524       .6000000   -.0000000
 .64     .4011884       .6400000   -.0000000
 .68     .5065084       .6800000   -.0000000
 .72     .6160861       .7200000    .0000000
 .76     .7229891       .7599999    .0000001
 .80     .8195326       .8000000    .0000000
 .84     .8984757       .8400000    .0000000
 .88     .9546199       .8800000    .0000000
 .92     .9866251       .9200000    .0000000
 .96     .9985659       .9600000    .0000000
1.00    1.0000000      1.0000000   0.0000000

A =     7.5 AND B =     3.5
  X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0000000 NOT  DEFINED NOT DEFINED
 .08     .0000004       .0800001   -.0000001
 .12     .0000073       .1200000   -.0000000
 .16     .0000571       .1600000   -.0000000
 .20     .0002744       .2000000   -.0000000
 .24     .0009662       .2400000   -.0000000
 .28     .0027404       .2799999    .0000001
 .32     .0066249       .3200000    .0000000
 .36     .0141509       .3600000   -.0000000
 .40     .0273677       .4000000   -.0000000
 .44     .0487533       .4400000    .0000000
 .48     .0810007       .4800000    .0000000
 .52     .1266730       .5200000   -.0000000
 .56     .1877447       .5600000    .0000000
 .60     .2650746       .6000000   -.0000000
 .64     .3578791       .6400000   -.0000000
 .68     .4633101       .6800000   -.0000000
 .72     .5762616       .7200000    .0000000
 .76     .6895432       .7600000    .0000000
 .80     .7945574       .8000000    .0000000
 .84     .8825820       .8400000    .0000000
 .88     .9466743       .8799999    .0000001
 .92     .9840316       .9200000    .0000000
 .96     .9982606       .9600000    .0000000
1.00    1.0000000      1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.0 AND B =    3.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000001      .0800000   -.0000000
 .12     .0000029      .1200001   -.0000001
 .16     .0000260      .1600000    .0000000
 .20     .0001395      .2000000   -.0000000
 .24     .0005375      .2400000   -.0000000
 .28     .0016447      .2800001   -.0000001
 .32     .0042448      .3200000    .0000000
 .36     .0096028      .3600000   -.0000000
 .40     .0195436      .4000000   -.0000000
 .44     .0364455      .4400000    .0000000
 .48     .0631079      .4800000    .0000000
 .52     .1024660      .5200000   -.0000000
 .56     .1571463      .5600000    .0000000
 .60     .2288871      .6000000   -.0000000
 .64     .3178877      .6400000   -.0000000
 .68     .4221910      .6800000    .0000000
 .72     .5372527      .7200000    .0000000
 .76     .6558826      .7600000    .0000000
 .80     .7687675      .8000000    .0000000
 .84     .8657638      .8400000    .0000000
 .88     .9380682      .8800000    .0000000
 .92     .9811592      .9200000    .0000000
 .96     .9979152      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000

A =    8.5 AND B =    3.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000011      .1200000    .0000000
 .16     .0000118      .1600000   -.0000000
 .20     .0000705      .2000000   -.0000000
 .24     .0002972      .2400000    .0000000
 .28     .0009811      .2800000   -.0000000
 .32     .0027039      .3200000    .0000000
 .36     .0064792      .3600000   -.0000000
 .40     .0138789      .4000000   -.0000000
 .44     .0270989      .4400000    .0000000
 .48     .0489148      .4800000   -.0000000
 .52     .0824791      .5200000   -.0000000
 .56     .1309266      .5600000    .0000000
 .60     .1957878      .6000000   -.0000000
 .64     .2812486      .6400000   -.0000000
 .68     .3833571      .6800000    .0000000
 .72     .4933415      .7200000    .0000000
 .76     .6222712      .7600000   -.0000000
 .80     .7423451      .8000000    .0000000
 .84     .8481068      .8400000    .0000000
 .88     .9288198      .8800000    .0000000
 .92     .9780030      .9200000    .0000000
 .96     .9975274      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    9.0 AND B =    3.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000004     .1200000     .0000000
 .16     .0000053     .1600000     .0000000
 .20     .0000354     .2000000    -.0000000
 .24     .0001634     .2400000     .0000000
 .28     .0005821     .2800000     .0000000
 .32     .0017131     .3200000     .0000000
 .36     .0043489     .3600000     .0000000
 .40     .0088063     .4000000    -.0000000
 .44     .0200506     .4400000     .0000000
 .48     .0377350     .4800000    -.0000000
 .52     .0660912     .5200000    -.0000000
 .56     .1086154     .5600000     .0000000
 .60     .1685133     .6000000    -.0000000
 .64     .2479152     .6400000    -.0000000
 .68     .3469377     .6800000     .0000000
 .72     .4627551     .7200000     .0000000
 .76     .5889439     .7600000     .0000000
 .80     .7154642     .8000000     .0000000
 .84     .8296990     .8400000     .0000000
 .88     .9189509     .8800000     .0000000
 .92     .9745591     .9200000     .0000000
 .96     .9970952     .9599999     .0000001
1.00    1.0000000    1.0000000    0.0000000

A =    4.0 AND B =    4.0
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000813     .0400000    -.0000000
 .08     .0011763     .0800000    -.0000000
 .12     .0053693     .1200000     .0000000
 .16     .0152503     .1600000    -.0000000
 .20     .0333440     .2000000    -.0000000
 .24     .0616955     .2400000     .0000000
 .28     .1015962     .2800000     .0000000
 .32     .1534344     .3200000    -.0000000
 .36     .2166517     .3600001    -.0000001
 .40     .2897920     .4000000    -.0000000
 .44     .3706237     .4399999     .0000001
 .48     .4563199     .4800000    -.0000000
 .52     .5436801     .5200000     .0000000
 .56     .6293763     .5600000     .0000000
 .60     .7102080     .6000000     .0000000
 .64     .7833483     .6400000     .0000000
 .68     .8465656     .6800000     .0000000
 .72     .8984038     .7200000     .0000000
 .76     .9383045     .7600000     .0000000
 .80     .9666560     .8000000     .0000000
 .84     .9847497     .8400000     .0000000
 .88     .9946307     .8800000     .0000000
 .92     .9988237     .9200000     .0000000
 .96     .9999187     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

 A =     4.5 AND B =    4.0
   X        BETA(X)         INVERSE XC   ERROR TERM
  .04      .0000207        .0400000    -.0000000
  .08      .0004228        .0800000    -.0000000
  .12      .0023572        .1200000     .0000000
  .16      .0077080        .1600000     .0000000
  .20      .0187815        .2000000    -.0000000
  .24      .0379325        .2400000     .0000000
  .28      .0672052        .2800000     .0000000
  .32      .1080324        .3200000    -.0000000
  .36      .1610168        .3600000     .0000000
  .40      .2258059        .4000000    -.0000000
  .44      .3010672        .4400000    -.0000000
  .48      .3845578        .4800000    -.0000000
  .52      .4732815        .5200000     .0000000
  .56      .5637198        .5600000     .0000000
  .60      .6521187        .6000000     .0000000
  .64      .7348104        .6400000     .0000000
  .68      .8085455        .6800000     .0000000
  .72      .8708067        .7200000     .0000000
  .76      .9200744        .7600000     .0000000
  .80      .9560096        .8000000     .0000000
  .84      .9795188        .8400000     .0000000
  .88      .9926624        .8799999     .0000001
  .92      .9983649        .9200000     .0000000
  .96      .9998851        .9600000     .0000000
 1.00     1.0000000       1.0000000    0.0000000

 A =     5.0 AND B =    4.0
   X        BETA(X)         INVERSE XC   ERROR TERM
  .04      .0000052        .0400000    -.0000000
  .08      .0001493        .0800001    -.0000001
  .12      .0010169        .1199999     .0000001
  .16      .0038303        .1600000     .0000000
  .20      .0104064        .2000000    -.0000000
  .24      .0229548        .2400000     .0000000
  .28      .0437826        .2800000    -.0000000
  .32      .0749644        .3200000    -.0000000
  .36      .1180242        .3600000     .0000000
  .40      .1736704        .4000000    -.0000000
  .44      .2416115        .4400000    -.0000000
  .48      .3204741        .4800001    -.0000001
  .52      .4078342        .5200000     .0000000
  .56      .5003641        .5600000     .0000000
  .60      .5940864        .6000000     .0000000
  .64      .6847209        .6400000     .0000000
  .68      .7680957        .6800000     .0000000
  .72      .8405901        .7199999     .0000001
  .76      .8995638        .7600000     .0000000
  .80      .9437184        .8000000     .0000000
  .84      .9733297        .8400000     .0000000
  .88      .9902784        .8800000     .0000000
  .92      .9977967        .9200000     .0000000
  .96      .9998426        .9600000     .0000000
 1.00     1.0000000       1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    5.5 AND B =    4.0
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000013     .0400000   -.0000000
 .08     .0000519     .0800000   -.0000000
 .12     .0004322     .1200000    .0000000
 .16     .0018757     .1600000    .0000000
 .20     .0056843     .2000000   -.0000000
 .24     .0137004     .2400000    .0000000
 .28     .0281456     .2800000   -.0000000
 .32     .0513568     .3200000   -.0000000
 .36     .0854611     .3600000    .0000000
 .40     .1320365     .4000000   -.0000000
 .44     .1918036     .4400000   -.0000000
 .48     .2643908     .4800000    .0000000
 .52     .3482078     .5200000    .0000000
 .56     .4404540     .5600000   -.0000000
 .60     .5372751     .6000000    .0000000
 .64     .6340695     .6400000    .0000000
 .68     .7259272     .6800000    .0000000
 .72     .8081721     .7199999    .0000001
 .76     .8769531     .7599999    .0000001
 .80     .9298150     .8000000    .0000000
 .84     .9661552     .8400000    .0000000
 .88     .9874494     .8800000    .0000000
 .92     .9971072     .9200000    .0000000
 .96     .9997899     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000

A =    6.0 AND B =    4.0
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000003     .0400000   -.0000000
 .08     .0000178     .0800000   -.0000000
 .12     .0001813     .1200000    .0000000
 .16     .0009068     .1600000    .0000000
 .20     .0030664     .2000000   -.0000000
 .24     .0080784     .2400000    .0000000
 .28     .0178821     .2800000    .0000000
 .32     .0347877     .3200000   -.0000000
 .36     .0612147     .3600000    .0000000
 .40     .0993526     .4000000   -.0000000
 .44     .1507869     .4400000   -.0000000
 .48     .2161445     .4800000    .0000000
 .52     .2948105     .5200000    .0000000
 .56     .3847691     .5600000   -.0000000
 .60     .4826097     .6000000   -.0000000
 .64     .5837263     .6400000    .0000000
 .68     .6827203     .6800000    .0000000
 .72     .7739888     .7200000    .0000000
 .76     .8524552     .7600000    .0000000
 .80     .9143583     .8000000    .0000000
 .84     .9579813     .8400000    .0000000
 .88     .9841503     .8800000    .0000000
 .92     .9962849     .9200000    .0000000
 .96     .9997257     .9599999    .0000001
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.5 AND B =    4.0
  X      BETA(X)      INVERSE XC  ERROR TERM
 .04    .0000001 NOT DEFINED NOT DEFINED
 .08    .0000060     .0800000   -.0000000
 .12    .0000752     .1200000    .0000000
 .16    .0004335     .1600000    .0000000
 .20    .0016360     .2000000   -.0000000
 .24    .0047125     .2400000    .0000000
 .28    .0112435     .2800000    .0000000
 .32    .0233281     .3200000   -.0000000
 .36    .0434246     .3600000    .0000000
 .40    .0740700     .4000000   -.0000000
 .44    .1175044     .4400000    .0000000
 .48    .1752488     .4800000    .0000000
 .52    .2476940     .5199999    .0000001
 .56    .3337729     .5600000   -.0000000
 .60    .4307838     .6000000   -.0000000
 .64    .5344275     .6400000    .0000000
 .68    .6391029     .6800000    .0000000
 .72    .7384769     .7200000    .0000000
 .76    .8263051     .7600000    .0000000
 .80    .8974290     .8000000    .0000000
 .84    .9488068     .8400000    .0000000
 .88    .9803598     .8800000    .0000000
 .92    .9953189     .9200000    .0000000
 .96    .9996487     .9599999    .0000001
1.00   1.0000000    1.0000000   0.0000000

A =    7.0 AND B =    4.0
  X      BETA(X)      INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000020     .0800000   -.0000000
 .12    .0000308     .1200000   -.0000000
 .16    .0002051     .1600000    .0000000
 .20    .0008644     .2000000   -.0000000
 .24    .0027228     .2399999    .0000001
 .28    .0070039     .2800000   -.0000000
 .32    .0155029     .3200000   -.0000000
 .36    .0305376     .3600000    .0000000
 .40    .0547619     .4000000   -.0000000
 .44    .0908427     .4400000    .0000000
 .48    .1410272     .4800000    .0000000
 .52    .2066520     .5200000   -.0000000
 .56    .2876693     .5600001   -.0000001
 .60    .3822806     .6000000   -.0000000
 .64    .4867716     .6400000    .0000000
 .68    .5956374     .6800000   -.0000000
 .72    .7020595     .7200000    .0000000
 .76    .7987513     .7600000    .0000000
 .80    .8791261     .8000000    .0000000
 .84    .9386423     .8400000    .0000000
 .88    .9760612     .8800000    .0000000
 .92    .9941987     .9200000    .0000000
 .96    .9995574     .9600000    .0000000
1.00   1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    7.5 AND B =    4.0
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000007     .0800001    -.0000001
 .12     .0000125     .1200000    -.0000000
 .16     .0000962     .1600000     .0000000
 .20     .0004527     .2000000    -.0000000
 .24     .0015598     .2400000    -.0000000
 .28     .0043266     .2800000     .0000000
 .32     .0102193     .3200001    -.0000001
 .36     .0213070     .3600000     .0000000
 .40     .0401818     .4000000    -.0000000
 .44     .0697243     .4400000     .0000000
 .48     .1127123     .4800000     .0000000
 .52     .1713036     .5200000    -.0000000
 .56     .2464585     .5600000     .0000000
 .60     .3373991     .6000000    -.0000000
 .64     .4412240     .6400000    -.0000000
 .68     .5523130     .6800000    -.0000000
 .72     .6651360     .7200000     .0000000
 .76     .7700469     .7600000     .0000000
 .80     .8595623     .8000000     .0000000
 .84     .9275084     .8400000     .0000000
 .88     .9712415     .8800000     .0000000
 .92     .9929144     .9200000     .0000000
 .96     .9994505     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000

A =    8.0 AND B =    4.0
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000002     .0800000    -.0000000
 .12     .0000051     .1200000    -.0000000
 .16     .0000448     .1600000    -.0000000
 .20     .0002352     .2000000    -.0000000
 .24     .0008867     .2400000    -.0000000
 .28     .0026526     .2800000    -.0000000
 .32     .0066870     .3200000     .0000000
 .36     .0147609     .3600000    -.0000000
 .40     .0292815     .4000000    -.0000000
 .44     .0531634     .4400000     .0000000
 .48     .0895181     .4800000     .0000000
 .52     .1411628     .5200000    -.0000000
 .56     .2099895     .5600000     .0000000
 .60     .2962843     .6000000    -.0000000
 .64     .3981272     .6400000    -.0000000
 .68     .5110427     .6800000    -.0000000
 .72     .6280749     .7200000     .0000000
 .76     .7404442     .7600000     .0000000
 .80     .8388608     .8000000     .0000000
 .84     .9154354     .8400000     .0000000
 .88     .9658920     .8800000     .0000000
 .92     .9914567     .9200000     .0000000
 .96     .9993266     .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     8.5 AND B =     4.0
  X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000001 NOT DEFINED NOT DEFINED
 .12     .0000020        .1200000      .0000000
 .16     .0000207        .1600000      .0000000
 .20     .0001213        .2000000     -.0000000
 .24     .0005005        .2400001     -.0000001
 .28     .0016152        .2800000      .0000000
 .32     .0043465        .3200000      .0000000
 .36     .0101597        .3600000     -.0000000
 .40     .0212045        .4000000     -.0000000
 .44     .0402918        .4400000      .0000000
 .48     .0706877        .4799999      .0000001
 .52     .1156913        .5200000     -.0000000
 .56     .1780042        .5600000      .0000000
 .60     .2589560        .6000000     -.0000000
 .64     .3577136        .6400000     -.0000000
 .68     .4706649        .6800000     -.0000000
 .72     .5912085        .7200000      .0000000
 .76     .7101882        .7600000      .0000000
 .80     .8171516        .8000000      .0000000
 .84     .9024614        .8399999      .0000001
 .88     .9600077        .8800000      .0000000
 .92     .9898173        .9200000      .0000000
 .96     .9991841        .9600000      .0000000
1.00    1.0000000       1.0000000     0.0000000

A =     9.0 AND B =     4.0
  X       BETA(X)         INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000000 NOT DEFINED NOT DEFINED
 .12     .0000008        .1200000      .0000000
 .16     .0000095        .1600000      .0000000
 .20     .0000622        .2000000     -.0000000
 .24     .0002807        .2400000      .0000000
 .28     .0009774        .2800000     -.0000000
 .32     .0028080        .3200000      .0000000
 .36     .0069513        .3600000     -.0000000
 .40     .0152673        .4000000     -.0000000
 .44     .0303675        .4400000      .0000000
 .48     .0555222        .4800000     -.0000000
 .52     .0943380        .5200000     -.0000000
 .56     .1501761        .5600000      .0000000
 .60     .2253373        .6000000     -.0000000
 .64     .3201202        .6400000     -.0000000
 .68     .4319466        .6800000      .0000000
 .72     .5548303        .7200000      .0000000
 .76     .6795133        .7600000      .0000000
 .80     .7945689        .8000000      .0000000
 .84     .8886315        .8400000      .0000000
 .88     .9535873        .8800000      .0000000
 .92     .9879882        .9200000      .0000000
 .96     .9990218        .9600000      .0000000
1.00    1.0000000       1.0000000     0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    4.5 AND B =     4.5
  X       BETA(X)        INVERSE XC   ERROR TERM
.04     .0000302      .0400000    -.0000000
.08     .0006058      .0800000    -.0000000
.12     .0033192      .1200000     .0000000
.16     .0106607      .1600000     .0000000
.20     .0255017      .2000000    -.0000000
.24     .0505405      .2400000     .0000000
.28     .0878253      .2800000     .0000000
.32     .1384093      .3200000    -.0000000
.36     .2021599      .3600000     .0000000
.40     .2777228      .4000000    -.0000000
.44     .3626291      .4400000    -.0000000
.48     .4535226      .4800000    -.0000000
.52     .5464774      .5200000     .0000000
.56     .6373709      .5600000     .0000000
.60     .7222772      .6000000     .0000000
.64     .7978401      .6400000     .0000000
.68     .8615907      .6800000     .0000000
.72     .9121747      .7200000     .0000000
.76     .9494595      .7600000     .0000000
.80     .9744983      .8000000     .0000000
.84     .9893393      .8400000     .0000000
.88     .9966808      .8800000     .0000000
.92     .9993942      .9200000     .0000000
.96     .9999698      .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000

A =    5.0 AND B =     4.5
  X       BETA(X)        INVERSE XC   ERROR TERM
.04     .0000078      .0400000    -.0000000
.08     .0002201      .0800001    -.0000001
.12     .0014731      .1199999     .0000001
.16     .0054472      .1600000     .0000000
.20     .0145216      .2000000    -.0000000
.24     .0314141      .2400000     .0000000
.28     .0587293      .2799999     .0000001
.32     .0985090      .3200000    -.0000000
.36     .1518561      .3600000     .0000000
.40     .2186823      .4000000    -.0000000
.44     .2976060      .4400000    -.0000000
.48     .3860105      .4800000    -.0000000
.52     .4802544      .5200000     .0000000
.56     .5760114      .5600000     .0000000
.60     .6687036      .6000000     .0000000
.64     .7539831      .6400000     .0000000
.68     .8282074      .6799999     .0000001
.72     .8888528      .7200000     .0000000
.76     .9348064      .7600000     .0000000
.80     .9664840      .8000000     .0000000
.84     .9857306      .8400000     .0000000
.88     .9954772      .8800000     .0000000
.92     .9991600      .9200000     .0000000
.96     .9999574      .9600000     .0000000
1.00    1.0000000    1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     5.5 AND B =     4.5
  X       BETA(X)         INVERSE XC  ERROR TERM
.04     .0000020        .0400000    -.0000000
.08     .0000786        .0800000    -.0000000
.12     .0006430        .1200000     .0000000
.16     .0027390        .1600001    -.0000001
.20     .0081412        .2000000    -.0000000
.24     .0192342        .2399999     .0000001
.28     .0387088        .2800000    -.0000000
.32     .0691489        .3200000    -.0000000
.36     .1125840        .3600000     .0000000
.40     .1700836        .4000000    -.0000000
.44     .2414569        .4400000    -.0000000
.48     .3251063        .4800000     .0000000
.52     .4180612        .5200000     .0000000
.56     .5161989        .5600000     .0000000
.60     .6146382        .6000000     .0000000
.64     .7082644        .6400000     .0000000
.68     .7923305        .6800000     .0000000
.72     .8630584        .7200000     .0000000
.76     .9181533        .7600000     .0000000
.80     .9571380        .8000000     .0000000
.84     .9814177        .8400000     .0000000
.88     .9940047        .8800000     .0000000
.92     .9988670        .9200000     .0000000
.96     .9999416        .9600000     .0000000
1.00   1.0000000       1.0000000    0.0000000

A =     6.0 AND B =     4.5
  X       BETA(X)         INVERSE XC  ERROR TERM
.04     .0000005        .0400000    -.0000000
.08     .0000277        .0800000    -.0000000
.12     .0002767        .1200000     .0000000
.16     .0013579        .1600000     .0000000
.20     .0045021        .2000000    -.0000000
.24     .0116214        .2400000     .0000000
.28     .0251885        .2800000     .0000000
.32     .0479466        .3200000    -.0000000
.36     .0824959        .3600000     .0000000
.40     .1308270        .4000000    -.0000000
.44     .1938778        .4400000    -.0000000
.48     .2711929        .4800000     .0000000
.52     .3607484        .5200000     .0000000
.56     .4589901        .5600000    -.0000000
.60     .5611032        .6000000     .0000000
.64     .6615029        .6400000     .0000000
.68     .7545007        .6800000     .0000000
.72     .8350678        .7200000     .0000000
.76     .8995850        .7600000     .0000000
.80     .9464452        .8000000     .0000000
.84     .9763610        .8400000     .0000000
.88     .9922374        .8800000     .0000000
.92     .9985074        .9200000     .0000000
.96     .9999217        .9600000     .0000000
1.00   1.0000000       1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.5 AND B =    4.5
  X       BETA(X)        INVERSE XC  ERROR TERM
.04      .0000001      .0400000   -.0000000
.08      .0000096      .0800000   -.0000000
.12      .0001175      .1200000    .0000000
.16      .0006649      .1600000    .0000000
.20      .0024597      .2000000   -.0000000
.24      .0069393      .2400000    .0000000
.28      .0162045      .2800000    .0000000
.32      .0328817      .3200000   -.0000000
.36      .0598158      .3600000    .0000000
.40      .0996289      .4000000   -.0000000
.44      .1542130      .4400001   -.0000001
.48      .2242411      .4800000    .0000000
.52      .3087906      .5200000    .0000000
.56      .4051612      .5600000   -.0000000
.60      .5089562      .6000000   -.0000000
.64      .6144541      .6400000    .0000000
.68      .7152626      .6800000    .0000000
.72      .8051903      .7199999    .0000001
.76      .8792197      .7600000    .0000000
.80      .9344117      .8000000    .0000000
.84      .9705290      .8400000    .0000000
.88      .9901511      .8800000    .0000000
.92      .9980733      .9200000    .0000000
.96      .9998973      .9600000    .0000000
1.00    1.0000000     1.0000000    0.0000000

A =    7.0 AND B =    4.5
  X       BETA(X)        INVERSE XC  ERROR TERM
.04      .0000000 NOT DEFINED NOT DEFINED
.08      .0000033      .0800000   -.0000000
.12      .0000494      .1200000   -.0000000
.16      .0003220      .1600000    .0000000
.20      .0013293      .2000000   -.0000000
.24      .0041001      .2400000    .0000000
.28      .0103187      .2800000    .0000000
.32      .0223283      .3200000   -.0000000
.36      .0429607      .3600000    .0000000
.40      .0751853      .4000000   -.0000000
.44      .1216138      .4400000    .0000000
.48      .1839314      .4800000    .0000000
.52      .2623551      .5199999    .0000001
.56      .3552313      .5600000   -.0000000
.60      .4588829      .6000000   -.0000000
.64      .5677897      .6400000   -.0000000
.68      .6751432      .6800000    .0000000
.72      .7737530      .7200000    .0000000
.76      .8572020      .7599999    .0000001
.80      .9210627      .8000000    .0000000
.84      .9638993      .8400000    .0000000
.88      .9877233      .8799999    .0000001
.92      .9975568      .9200000    .0000000
.96      .9998675      .9600000    .0000000
1.00    1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     7.5 AND B =     4.5
   X      BETA(X)         INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000011       .0800001    -.0000001
 .12     .0000205       .1200000    -.0000000
 .16     .0001544       .1600000    -.0000000
 .20     .0007115       .2000000    -.0000000
 .24     .0023997       .2400000    -.0000000
 .28     .0065104       .2800000    -.0000000
 .32     .0150271       .3200001    -.0000001
 .36     .0305903       .3600000     .0000000
 .40     .0562722       .4000000    -.0000000
 .44     .0951556       .4400000     .0000000
 .48     .1497561       .4800000     .0000000
 .52     .2213741       .5200000    -.0000000
 .56     .3094980       .5600001    -.0000001
 .60     .4114036       .6000000    -.0000000
 .64     .5220871       .6400000     .0000000
 .68     .6346378       .6799999     .0000001
 .72     .7410902       .7200000     .0000000
 .76     .8336973       .7600000     .0000000
 .80     .9064408       .8000000     .0000000
 .84     .9564575       .8400000     .0000000
 .88     .9849339       .8799999     .0000001
 .92     .9969500       .9200000     .0000000
 .96     .9998318       .9600000     .0000000
1.00    1.0000000      1.0000000    0.0000000

A =     8.0 AND B =     4.5
   X      BETA(X)         INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000004       .0800000    -.0000000
 .12     .0000085       .1200000    -.0000000
 .16     .0000734       .1600000     .0000000
 .20     .0003775       .2000000    -.0000000
 .24     .0013925       .2400000    -.0000000
 .28     .0040734       .2800000     .0000000
 .32     .0100315       .3200000     .0000000
 .36     .0216116       .3600000    -.0000000
 .40     .0418003       .4000000    -.0000000
 .44     .0739195       .4400000     .0000000
 .48     .1211032       .4800000     .0000000
 .52     .1856083       .5200000    -.0000000
 .56     .2680739       .5600000     .0000000
 .60     .3668846       .6000000    -.0000000
 .64     .4778250       .6399999     .0000001
 .68     .5941985       .6800000    -.0000000
 .72     .7075329       .7200000     .0000000
 .76     .8088854       .7600000     .0000000
 .80     .8906036       .8000000     .0000000
 .84     .9481976       .8400000     .0000000
 .88     .9817647       .8800000     .0000000
 .92     .9962450       .9200000     .0000000
 .96     .9997894       .9600000     .0000000
1.00    1.0000000      1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.5 AND B =    4.5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000001    .0800000    -.0000000
 .12    .0000035    .1200000     .0000000
 .16    .0000346    .1600000     .0000000
 .20    .0001987    .2000000    -.0000000
 .24    .0008018    .2400000    -.0000000
 .28    .0025293    .2800000     .0000000
 .32    .0066473    .3200000     .0000000
 .36    .0151592    .3600000    -.0000000
 .40    .0308362    .4000000    -.0000000
 .44    .0570439    .4400000     .0000000
 .48    .0973187    .4800000     .0000000
 .52    .1547045    .5200000    -.0000000
 .56    .2309266    .5600000     .0000000
 .60    .3255573    .6000000    -.0000000
 .64    .4353853    .6400000    -.0000000
 .68    .5512280    .6800001    -.0000001
 .72    .6734006    .7200000     .0000000
 .76    .7829551    .7600000     .0000000
 .80    .8736217    .8000000     .0000000
 .84    .9391214    .8400000     .0000000
 .88    .9782002    .8800000     .0000000
 .92    .9954342    .9200000     .0000000
 .96    .9997396    .9600000     .0000000
1.00   1.0000000   1.0000000    0.0000000

A =    9.0 AND B =    4.5
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000000 NOT DEFINED NOT DEFINED
 .12    .0000014    .1200000     .0000000
 .16    .0000162    .1600000    -.0000000
 .20    .0001038    .2000000    -.0000000
 .24    .0004584    .2400000     .0000000
 .28    .0015597    .2800000     .0000000
 .32    .0043750    .3200000     .0000000
 .36    .0105635    .3600000    -.0000000
 .40    .0226039    .4000000    -.0000000
 .44    .0437529    .4400000     .0000000
 .48    .0777517    .4799999     .0000001
 .52    .1252401    .5200000    -.0000000
 .56    .1979122    .5600000     .0000000
 .60    .2875361    .6000000    -.0000000
 .64    .3950574    .6400000    -.0000000
 .68    .5150750    .6800000    -.0000000
 .72    .6389951    .7200000     .0000000
 .76    .7560994    .7600000     .0000000
 .80    .8555758    .8000000     .0000000
 .84    .9292379    .8400000     .0000000
 .88    .9742270    .8800000     .0000000
 .92    .9945101    .9200000     .0000000
 .96    .9996817    .9600000     .0000000
1.00   1.0000000   1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     5.0 AND B =    5.0
  X        BETA(X)        INVERSE XC   ERROR TERM
 .04      .0000113      .0400000     -.0000000
 .08      .0003136      .0800001     -.0000001
 .12      .0020615      .1200000      .0000000
 .16      .0074847      .1600000      .0000000
 .20      .0195814      .2000000     -.0000000
 .24      .0415503      .2400000      .0000000
 .28      .0761583      .2800000      .0000000
 .32      .1251852      .3200000     -.0000000
 .36      .1890360      .3600000      .0000000
 .40      .2665677      .4000000     -.0000000
 .44      .3551423      .4400000     -.0000000
 .48      .4508861      .4800000     -.0000000
 .52      .5491139      .5200000      .0000000
 .56      .6448577      .5600000      .0000000
 .60      .7334323      .6000000      .0000000
 .64      .8109640      .6400000      .0000000
 .68      .8748148      .6800000      .0000000
 .72      .9238417      .7200000      .0000000
 .76      .9584497      .7600000      .0000000
 .80      .9804186      .8000000      .0000000
 .84      .9925153      .8400000      .0000000
 .88      .9979385      .8800000      .0000000
 .92      .9996864      .9199999      .0000001
 .96      .9999887      .9600000      .0000000
1.00     1.0000000     1.0000000     0.0000000

A =     5.5 AND B =    5.0
  X        BETA(X)        INVERSE XC   ERROR TERM
 .04      .0000029      .0400000     -.0000000
 .08      .0001149      .0800000     -.0000000
 .12      .0009230      .1200000      .0000000
 .16      .0038587      .1600000     -.0000000
 .20      .0112506      .2000000     -.0000000
 .24      .0260588      .2399999      .0000001
 .28      .0513861      .2800000     -.0000000
 .32      .0898962      .3200000     -.0000000
 .36      .1432612      .3600000      .0000000
 .40      .2117404      .4000000     -.0000000
 .44      .2939650      .4400000     -.0000000
 .48      .3869611      .4800000     -.0000000
 .52      .4864143      .5200000      .0000000
 .56      .5871404      .5600000      .0000000
 .60      .6837007      .6000000      .0000000
 .64      .7710771      .6400000      .0000000
 .68      .8453106      .6800000      .0000000
 .72      .9040031      .7200000      .0000000
 .76      .9465941      .7600000      .0000000
 .80      .9743458      .8000000      .0000000
 .84      .9900092      .8400000      .0000000
 .88      .9971976      .8800000      .0000000
 .92      .9995661      .9199999      .0000001
 .96      .9999841      .9600000      .0000000
1.00     1.0000000     1.0000000     0.0000000
```

```
  INCOMPLETE BETA FUNCTION AND INVERSE

   A =    6.0 AND B =     5.0
     X        BETA(X)        INVERSE XC  ERROR TERM
   .04      .0000007      .0400000    -.0000000
   .08      .0000415      .0800000    -.0000000
   .12      .0004069      .1200000     .0000000
   .16      .0019593      .1600000     .0000000
   .20      .0063694      .2000000    -.0000000
   .24      .0161116      .2400000     .0000000
   .28      .0341994      .2800000    -.0000000
   .32      .0637149      .3200000    -.0000000
   .36      .1072304      .3600000     .0000000
   .40      .1662386      .4000000    -.0000000
   .44      .2407033      .4400000    -.0000000
   .48      .3288205      .4800000     .0000000
   .52      .4270483      .5200000     .0000000
   .56      .5304187      .5600000    -.0000000
   .60      .6331033      .6000000     .0000000
   .64      .7291585      .6400000     .0000000
   .68      .8133446      .6799999     .0000001
   .72      .8818829      .7200000     .0000000
   .76      .9330110      .7600000     .0000000
   .80      .9672065      .8000000     .0000000
   .84      .9869899      .8400000     .0000000
   .88      .9962839      .8800000     .0000000
   .92      .9994143      .9199999     .0000001
   .96      .9999782      .9600000     .0000000
  1.00     1.0000000     1.0000000    0.0000000

   A =    6.5 AND B =     5.0
     X        BETA(X)        INVERSE XC  ERROR TERM
   .04      .0000002      .0400000    -.0000000
   .08      .0000147      .0800000    -.0000000
   .12      .0001769      .1200000     .0000000
   .16      .0009815      .1600000     .0000000
   .20      .0035589      .2000000    -.0000000
   .24      .0098356      .2400000     .0000000
   .28      .0224834      .2800000     .0000000
   .32      .0446299      .3200000    -.0000000
   .36      .0793658      .3600000     .0000000
   .40      .1291382      .4000000    -.0000000
   .44      .1951471      .4400000    -.0000000
   .48      .2768707      .4800000     .0000000
   .52      .3718286      .5200000     .0000000
   .56      .4756587      .5600000    -.0000000
   .60      .5825339      .6000000     .0000000
   .64      .6858833      .6399999     .0000001
   .68      .7793242      .6800000     .0000000
   .72      .8576558      .7200000     .0000000
   .76      .9177247      .7600000     .0000000
   .80      .9589624      .8000000     .0000000
   .84      .9834168      .8400000     .0000000
   .88      .9951771      .8800000     .0000000
   .92      .9992263      .9200000     .0000000
   .96      .9999707      .9600000     .0000000
  1.00     1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     7.0 AND B =     5.0
  X        BETA(X)        INVERSE XC   ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000052      .0800000    -.0000000
 .12      .0000760      .1200000    -.0000000
 .16      .0004858      .1600000     .0000000
 .20      .0019654      .2000000    -.0000000
 .24      .0059361      .2400000     .0000000
 .28      .0146197      .2800000     .0000000
 .32      .0309308      .3200000    -.0000000
 .36      .0581470      .3600000     .0000000
 .40      .0993526      .4000000    -.0000000
 .44      .1567813      .4400001    -.0000001
 .48      .2311679      .4800000     .0000000
 .52      .3212580      .5200000     .0000000
 .56      .4236090      .5600000    -.0000000
 .60      .5327742      .6000000     .0000000
 .64      .6418992      .6400000     .0000000
 .68      .7436783      .6800000     .0000000
 .72      .8315324      .7200000     .0000000
 .76      .9007887      .7600000     .0000000
 .80      .9495904      .8000000     .0000000
 .84      .9792542      .8400000     .0000000
 .88      .9938572      .8800000     .0000000
 .92      .9989970      .9200000     .0000000
 .96      .9999614      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000

A =     7.5 AND B =     5.0
  X        BETA(X)        INVERSE XC   ERROR TERM
 .04      .0000000 NOT DEFINED NOT DEFINED
 .08      .0000018      .0800001    -.0000001
 .12      .0000323      .1200000    -.0000000
 .16      .0002379      .1600000     .0000000
 .20      .0010739      .2000000    -.0000000
 .24      .0035460      .2400000    -.0000000
 .28      .0094105      .2800000    -.0000000
 .32      .0212307      .3200000    -.0000000
 .36      .0422081      .3600000     .0000000
 .40      .0757644      .4000000    -.0000000
 .44      .1249104      .4400000     .0000000
 .48      .1915083      .4800000     .0000000
 .52      .2755766      .5199999     .0000001
 .56      .3748107      .5600000    -.0000000
 .60      .4844800      .6000000    -.0000000
 .64      .5978059      .6400000     .0000000
 .68      .7068406      .6800000     .0000000
 .72      .8037502      .7199999     .0000001
 .76      .8822820      .7600000     .0000000
 .80      .9390824      .8000000     .0000000
 .84      .9744715      .8400000     .0000000
 .88      .9923048      .8800000     .0000000
  92      .9987213      .9200000     .0000000
 .96      .9999499      .9600000     .0000000
1.00     1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =     8.0 AND B =     5.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000006      .0800000    -.0000000
 .12     .0000136      .1200000    -.0000000
 .16     .0001153      .1600000     .0000000
 .20     .0005812      .2000000    -.0000000
 .24     .0020985      .2400000    -.0000000
 .28     .0060031      .2800000     .0000000
 .32     .0144450      .3200000     .0000000
 .36     .0303799      .3600000     .0000000
 .40     .0573099      .4000000    -.0000000
 .44     .0987553      .4400000     .0000000
 .48     .1575100      .4800000     .0000000
 .52     .2348123      .5200000    -.0000000
 .56     .3296164      .5600000    -.0000000
 .60     .4381782      .6000000    -.0000000
 .64     .5541413      .6400000    -.0000000
 .68     .6692349      .6799999     .0000001
 .72     .7745643      .7200000     .0000000
 .76     .8623060      .7600000     .0000000
 .80     .9274445      .8000000     .0000000
 .84     .9690432      .8400000     .0000000
 .88     .9905014      .8800000     .0000000
 .92     .9983938      .9200000     .0000000
 .96     .9999360      .9600000     .0000000
1.00    1.0000000     1.0000000    0.0000000

A =     8.5 AND B =     5.0
  X       BETA(X)        INVERSE XC   ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000002      .0800000    -.0000000
 .12     .0000057      .1200000     .0000000
 .16     .0000554      .1600000     .0000000
 .20     .0003119      .2000000    -.0000000
 .24     .0012314      .2400000    -.0000000
 .28     .0037979      .2800000    -.0000000
 .32     .0097495      .3200000     .0000000
 .36     .0216971      .3600000    -.0000000
 .40     .0430284      .4000000    -.0000000
 .44     .0775240      .4400000     .0000000
 .48     .1286816      .4800000     .0000000
 .52     .1988316      .5200000    -.0000000
 .56     .2882158      .5600000     .0000000
 .60     .3942704      .6000000    -.0000000
 .64     .5113727      .6400000     .0000000
 .68     .6312644      .6799999     .0000001
 .72     .7442385      .7200000     .0000000
 .76     .8409795      .7599999     .0000001
 .80     .9146962      .8000000     .0000000
 .84     .9629499      .8400000     .0000000
 .88     .9884292      .8800000     .0000000
 .92     .9980090      .9200000     .0000000
 .96     .9999193      .9600000     .0000000
1.00    1.0000000     1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    9.0 AND B =    5.0
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000001 NOT DEFINED NOT DEFINED
 .12     .0000023    .1200000     .0000000
 .16     .0000264    .1600000     .0000000
 .20     .0001660    .2000000    -.0000000
 .24     .0007170    .2400000     .0000000
 .28     .0023846    .2800000    -.0000000
 .32     .0065319    .3200000     .0000000
 .36     .0153856    .3600000    -.0000000
 .40     .0320843    .4000000    -.0000000
 .44     .0604581    .4400000     .0000000
 .48     .1044764    .4800000     .0000000
 .52     .1673846    .5200000    -.0000000
 .56     .2506627    .5600000     .0000000
 .60     .3530418    .6000000    -.0000000
 .64     .4698937    .6400000    -.0000000
 .68     .5933026    .6800000    -.0000000
 .72     .7130388    .7200000     .0000000
 .76     .8184357    .7600000     .0000000
 .80     .9008694    .8000000     .0000000
 .84     .9561774    .8400000     .0000000
 .88     .9860717    .8800000     .0000000
 .92     .9975614    .9200000     .0000000
 .96     .9998994    .9600000     .0000000
1.00    1.0000000   1.0000000    0.0000000

A =    5.5 AND B =    5.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04     .0000042    .0400000    -.0000000
 .08     .0001630    .0800000    -.0000000
 .12     .0012853    .1200000     .0000000
 .16     .0052740    .1600000    -.0000000
 .20     .0150855    .2000000    -.0000000
 .24     .0342613    .2400000     .0000000
 .28     .0662141    .2800000    -.0000000
 .32     .1134756    .3200000    -.0000000
 .36     .1770787    .3600000     .0000000
 .40     .2561949    .4000000    -.0000000
 .44     .3480884    .4400000    -.0000000
 .48     .4483860    .4800000    -.0000000
 .52     .5516140    .5200000     .0000000
 .56     .6519116    .5599999     .0000001
 .60     .7438051    .6000000     .0000000
 .64     .8229213    .6400000     .0000000
 .68     .8865244    .6800000     .0000000
 .72     .9337859    .7200000     .0000000
 .76     .9657387    .7600000     .0000000
 .80     .9849145    .8000000     .0000000
 .84     .9947260    .8400000     .0000000
 .88     .9987147    .8800000     .0000000
 .92     .9998370    .9200000     .0000000
 .96     .9999958    .9600000     .0000000
1.00    1.0000000   1.0000000    0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    6.0 AND B =    5.5
  X      BETA(X)       INVERSE XC  ERROR TERM
.04     .0000011     .0400000   -.0000000
.08     .0000602     .0800000   -.0000000
.12     .0005798     .1200000    .0000000
.16     .0027392     .1600000    .0000000
.20     .0087326     .2000000   -.0000000
.24     .0216497     .2400000    .0000000
.28     .0450148     .2800000   -.0000000
.32     .0821043     .3200000   -.0000000
.36     .1352096     .3600000    .0000000
.40     .2050159     .4000000   -.0000000
.44     .2902298     .4400000   -.0000000
.48     .3875414     .4800000   -.0000000
.52     .4919394     .5200000    .0000000
.56     .5973352     .5600000    .0000000
.60     .6973970     .6000000    .0000000
.64     .7864539     .6400000    .0000000
.68     .8603108     .6800000    .0000000
.72     .9168210     .7200000    .0000000
.76     .9560958     .7600000    .0000000
.80     .9802886     .8000000    .0000000
.84     .9929765     .8399999    .0000001
.88     .9982562     .8799999    .0000001
.92     .9997748     .9200000    .0000000
.96     .9999941     .9599999    .0000001
1.00   1.0000000    1.0000000   0.0000000

A =    6.5 AND B =    5.5
  X      BETA(X)       INVERSE XC  ERROR TERM
.04     .0000003     .0400000   -.0000000
.08     .0000219     .0800000   -.0000000
.12     .0002577     .1200000    .0000000
.16     .0014024     .1600000   -.0000000
.20     .0049848     .2000000   -.0000000
.24     .0134966     .2400000    .0000000
.28     .0302072     .2800000    .0000000
.32     .0586717     .3200000   -.0000000
.36     .1020304     .3600000    .0000000
.40     .1622553     .4000000   -.0000000
.44     .2335182     .4400000   -.0000000
.48     .3318306     .4800000    .0000000
.52     .4350588     .5200000    .0000000
.56     .5433415     .5600000   -.0000000
.60     .6498656     .6000000    .0000000
.64     .7478732     .6400000    .0000000
.68     .8317207     .6800000    .0000000
.72     .8977792     .7200000    .0000000
.76     .9449742     .7600000    .0000000
.80     .9748140     .8000000    .0000000
.84     .9908546     .8399999    .0000001
.88     .9976871     .8800000    .0000000
.92     .9996959     .9200000    .0000000
.96     .9999918     .9599999    .0000001
1.00   1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    7.0 AND B =    5.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000001 NOT DEFINED NOT DEFINED
 .08     .0000079      .0800000   -.0000000
 .12     .0001130      .1200000   -.0000000
 .16     .0007087      .1600000   -.0000000
 .20     .0028100      .2000000   -.0000000
 .24     .0083120      .2400000    .0000000
 .28     .0200336      .2800000    .0000000
 .32     .0414566      .3200000   -.0000000
 .36     .0761703      .3600000    .0000000
 .40     .1271175      .4000000   -.0000000
 .44     .1958048      .4400000   -.0000000
 .48     .2816642      .4800000    .0000000
 .52     .3817390      .5200000    .0000000
 .56     .4908095      .5600000   -.0000000
 .60     .6019914      .6000000    .0000000
 .64     .7077348      .6400000    .0000000
 .68     .8010572      .6799999    .0000001
 .72     .8767620      .7200000    .0000000
 .76     .9323613      .7600000    .0000000
 .80     .9684434      .8000000    .0000000
 .84     .9883241      .8400000    .0000000
 .88     .9969923      .8800000    .0000000
 .92     .9995974      .9200000    .0000000
 .96     .9999890      .9599999    .0000001
1.00    1.0000000     1.0000000   0.0000000

A =    7.5 AND B =    5.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000028      .0800001   -.0000001
 .12     .0000490      .1200000   -.0000000
 .16     .0003541      .1600000    .0000000
 .20     .0015661      .2000000   -.0000000
 .24     .0050630      .2399999    .0000001
 .28     .0131455      .2800000   -.0000000
 .32     .0289934      .3200000   -.0000000
 .36     .0563087      .3600000    .0000000
 .40     .0986654      .4000000   -.0000000
 .44     .1586751      .4400001   -.0000001
 .48     .2371519      .4800000    .0000000
 .52     .3324902      .5200000    .0000000
 .56     .4404503      .5600000   -.0000000
 .60     .5544808      .6000000    .0000000
 .64     .6665902      .6400000    .0000000
 .68     .7686542      .6800000    .0000000
 .72     .8539063      .7200000    .0000000
 .76     .9182678      .7600000    .0000000
 .80     .9611394      .8000000    .0000000
 .84     .9853511      .8400000    .0000000
 .88     .9961567      .8800000    .0000000
 .92     .9994762      .9200000    .0000000
 .96     .9999854      .9600000    .0000000
1.00    1.0000000     1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    8.0 AND B =    5.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000010     .0800000   -.0000000
 .12     .0000210     .1200000   -.0000000
 .16     .0001750     .1600000   -.0000000
 .20     .0008640     .2000000   -.0000000
 .24     .0030532     .2400000   -.0000000
 .28     .0085423     .2800000    .0000000
 .32     .0200876     .3200001   -.0000001
 .36     .0412527     .3600000    .0000000
 .40     .0759272     .4000000   -.0000000
 .44     .1275491     .4400000    .0000000
 .48     .1981725     .4799999    .0000001
 .52     .2875963     .5199999    .0000001
 .56     .3928058     .5600000   -.0000000
 .60     .5079487     .6000000   -.0000000
 .64     .6249673     .6400000    .0000000
 .68     .7348624     .6800000    .0000000
 .72     .8293778     .7200000    .0000000
 .76     .9027270     .7600000    .0000000
 .80     .9528754     .8000000    .0000000
 .84     .9819039     .8400000    .0000000
 .88     .9951650     .8800000    .0000000
 .92     .9993292     .9200000    .0000000
 .96     .9999810     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000

A =    8.5 AND B =    5.5
  X       BETA(X)        INVERSE XC  ERROR TERM
 .04     .0000000 NOT DEFINED NOT DEFINED
 .08     .0000003     .0800000   -.0000000
 .12     .0000089     .1200000    .0000000
 .16     .0000857     .1600000   -.0000000
 .20     .0004722     .2000000   -.0000000
 .24     .0018245     .2400000   -.0000000
 .28     .0055018     .2800000    .0000000
 .32     .0137980     .3200000    .0000000
 .36     .0299730     .3600000   -.0000000
 .40     .0579679     .4000000   -.0000000
 .44     .1017616     .4400000    .0000000
 .48     .1644386     .4800000    .0000000
 .52     .2471531     .5200000   -.0000000
 .56     .3482599     .5600000   -.0000000
 .60     .4629114     .6000000   -.0000000
 .64     .5833564     .6400000    .0000000
 .68     .7000378     .6800000    .0000000
 .72     .8033647     .7200000    .0000000
 .76     .8857927     .7600000    .0000000
 .80     .9436358     .8000000    .0000000
 .84     .9779543     .8400000    .0000000
 .88     .9940019     .8800000    .0000000
 .92     .9991529     .9200000    .0000000
 .96     .9999756     .9600000    .0000000
1.00    1.0000000    1.0000000   0.0000000
```

```
INCOMPLETE BETA FUNCTION AND INVERSE

A =    9.0 AND B =    5.5
  X       BETA(X)       INVERSE XC  ERROR TERM
 .04    .0000000 NOT DEFINED NOT DEFINED
 .08    .0000001     .0800000    -.0000000
 .12    .0000038     .1200000     .0000000
 .16    .0000416     .1600000     .0000000
 .20    .0002559     .2000000    -.0000000
 .24    .0010811     .2400001    -.0000001
 .28    .0035148     .2800000     .0000000
 .32    .0094030     .3200000     .0000000
 .36    .0216116     .3600000    -.0000000
 .40    .0439332     .4000000    -.0000000
 .44    .0806232     .4400000     .0000000
 .48    .1355537     .4800000     .0000000
 .52    .2111053     .5200000    -.0000000
 .56    .3070526     .5600000     .0000000
 .60    .4197835     .6000000    -.0000000
 .64    .5421997     .6400000    -.0000000
 .68    .6645304     .6800000    -.0000000
 .72    .7760707     .7199999     .0000001
 .76    .8675364     .7600000     .0000000
 .80    .9334155     .8000000     .0000000
 .84    .9734773     .8400000     .0000000
 .88    .9926524     .8800000     .0000000
 .92    .9989437     .9199999     .0000001
 .96    .9999690     .9600000     .0000000
1.00   1.0000000    1.0000000    0.0000000
```

## PEARSON'S INCOMPLETE GAMMA FUNCTION

In the field of hydrology the incomplete gamma function is an extensively used tool for the analyses of flood frequencies and also in the study of time series.

An excellent discussion of the use of the incomplete gamma function as related to flood frequency analyses is found in the American Society of Civil Engineers Transactions, Volume 87, Paper No. 1532, by H. Alden Foster, dated 1924.

The use of the incomplete gamma function can be found in various papers concerning stochastic techniques in time series analyses.

The discussion that follows in the subsequent narrative will be limited to an electronic computer application for the solution of the incomplete gamma function.

The expression generally given for Pearson's Incomplete Gamma Function is as follows:

$$I(u,p) = \frac{1}{\Gamma(p+1)} \int_{0}^{u\sqrt{p+1}} t^{p}\ell^{-t}\,dt = P(x^{2}/v)$$

where: $x = u\sqrt{p+1}$

$a = p+1$ or $p = a-1$

The expression of the probability function for the solution of a random variable having a value $\leq \chi$ from a gamma function is:

$$P(x \leq \chi) = \frac{b^a}{\Gamma(a)} \int_o^{\chi} t^{a-1} \, \ell^{-bt} \, dt, \; x \geq 0$$

$$= | - \frac{1}{\Gamma(a)} \int_{\chi}^{\infty} \ell^{-t} \, t^{a-1} \, dt$$

which is often written as:

$$P(x \leq \chi) = |- \frac{1}{p(a)} \, r(a, \, b-\chi)$$

where "a" and "b" are distribution parameters and r is the incomplete gamma function.

$\Gamma(a)$ is the complete gamma function.

The purpose of this electronic computer application is to design a portable program for the calculation of the incomplete gamma function under five conditions (five algorithms).

$$r(A,x) = \Gamma(A,x) = \int_{\chi}^{\infty} \ell^{-u} \, \mu^{A-1} \, du$$

Condition 1 - if A = 0

then:

$$r(0.x) = \Gamma(0.x) = - \left( \gamma + \ell n \, (x) + \sum_{n=o}^{\infty} \frac{(-x)^n}{n.n!} \right)$$

Condition 2 - if A = -N for some positive integer N

then:

$$r(-N,x) = \Gamma(-N,x) = \frac{(-1)^N}{N!}\left(E_1(x) - \ell^{-x} \sum_{j=0}^{N-1} \frac{(-1)^j j!}{x^{j+1}}\right)$$

Condition 3 - if x = 0

$$r(A,0) = \Gamma(A,0) = \int_0^\infty \ell^{-u}\, u^{A-1}\, du = \Gamma(A)$$

Condition 4 - if $A \neq 0$ and $x < \sqrt{|A| + 1}$

$$r(A,x) = \Gamma(A,x) = \Gamma(A) - X^A \sum_{n=0}^{\infty} \frac{(-x)^n}{(A+n)n!}$$

Condition 5 - if $A \neq 0$ $X \geq \sqrt{|A| + 1}$

$$r(A,x) = \ell^{-x}\, X^A \quad \frac{1}{X+} \quad \frac{1 - A}{1+} \quad \frac{2 - A}{1+} \quad \frac{2}{X+} \cdots$$

The electronic computer application is a stand alone set of subroutines or functions. The discussion of these various subroutines follows in the subsequent narrative.

PROGRAM GAMINT

This program is simply a little driver that was used to generate values of Pearson's Incomplete Gamma Function in terms of "u" and "p" for comparison with the Tables.

The variable name A is A in $\Gamma(A,x)$ and the variable name X is X in $\Gamma(A,x)$. The variable name U is the transform $X = U\sqrt{P+1}$ and the variable name P is the transform $P = A-1$.

The variable name COX is $\Gamma(A)$ and CONGAM(A) is the function name used for the solution of the complete gamma.

FUNCTION GAMMA (A,X)

In the function labeled GAMMA the arguments A and X are identical to those described in the previous narrative concerning the five conditions.

The label EULERS is used to store Euler's constant, which is $\gamma$ is the previous narrative.

It is in the function (FORTRAN terminology) that the various tests are made to determine which condition of algorithm will be used in the solution of the gamma distribution.

Several comments have been included to aid in describing the aforementioned tests.

FUNCTION SMLGAM (A,X)

The function labeled SMLGAM is used to calculate the sum of the following series:

$$S(a,x) = \sum_{n=o}^{n=k} \frac{(-x)^n}{(a+n)n!}$$

where k = 30 provided enough terms in the series such that:

$$\left| S_n(a,x) - S_{n+1}(a,x) \right| \leq 1 \times 10^{-15}$$

This series expansion was used for the solution of Condition 1, i.e. Γ(0,X). The series expansion was also used in the solutions of Conditions 2 and 4.

The arguments labeled A and X are self-explanatory.

The label EPSLON represents the absolute difference of the sums n and n+1 in the criteria for the series truncation.

All other labels included in this function labeled SMLGAM are by design self-explanatory.

The series is shown in Reference 1, Chapter 6, Series Developments - 6.5.29, page 262.

FUNCTION CONTER (A,X)

The function labeled CONTER is used for the solution of the following continued fraction:

$$\frac{1}{X+}\ \frac{1-A}{1+}\ \frac{2-A}{1+}\ \frac{2}{X+}\ \ldots$$

where: $X > 0$ and $|A| < \infty$

This series is used as part of the solution for Condition 5. All labels have been designed to be self-explanatory.

The continued fraction is shown in Reference 1, Chapter 6, 6.5.31, page 263.

The arguments A and X are self-explanatory.

FUNCTION CONGAM (ALF)

The function labeled CONGAM is used for the solution of the complete gamma which is often called Euler's Integral. The expression for the complete gamma is as follows:

$$\Gamma(Z) = \int_{0}^{\infty} t^{Z-1}\ \ell^{-t}\ dt$$

The recurrence formulas used in the solution of the complete gamma are:

$$\Gamma(Z+1) = Z\Gamma(Z) = Z! + Z(Z-1)!$$

$$\Gamma(n+Z) = (n - 1 + Z)\ (n - 2 + Z)\ \ldots\ (1+Z)Z!$$

The reflection formula used in the solution of the complete gamma is:

$$\Gamma(Z)\Gamma(1-Z) = -Z\Gamma(-Z)\Gamma(Z) = \Pi \text{ CSC } \Pi Z$$

where the fundamental identity for CSC $\Pi Z = 1/\sin \Pi Z$.

The following polynomial approximation along with the above recurrence and reflection formulas are shown in Reference 1, Chapter 6, pages 256 and 257.

$$\Gamma(x+1) = X! = 1 + b_1 x + b_2 X^2 + \ldots + b_8 X^8 + \varepsilon(x)$$

where: $0 \leq X \leq 1$ and $|\varepsilon(x)| \leq 3 \times 10^{-7}$

The coefficients for the above polynomial are:

$b_1 = -0.577191652$

$b_2 = 0.988205891$

$b_3 = -0.897056937$

$b_4 = 0.918206857$

$b_5 = -0.756704078$

$b_6 = 0.482199394$

$b_7 = -0.193527818$

$b_8 = 0.035868343$

All labels are self-explanatory as well as the argument ALF.

## FUNCTION GAMNEG (MINUSN,X)

The function GAMNEG is used for the solution of Condition 2. It might be well to reiterate the expression for Condition 2.

$$\Gamma(-N,X) = {}^{(-1)^N}/_{N!} \left[ E_1(X) - \ell^{-x} \sum_{j=o}^{N-1} \frac{(-1)^j \, j!}{X^{j+1}} \right]$$

where: $$E_n(X) = \int_1^{\infty} \ell^{-xt} \, t^{-n} \, dt = X^{n-1} \, \Gamma(1-n,x)$$

or:

$$E_1(X) = \int_1^{\infty} \ell^{-xt} \, t^{-1} \, dt = X^{\circ} \, \Gamma(0,x) = \Gamma(X)$$

which is easily recognized as the complete gamma function.

The argument labeled MINUSN is A = -N for some positive integer N.

The expressions as shown above can be found in Reference 1, Chapter 6.

All labels used in the function labeled GAMNEG are self-explanatory.

## Error Messages

The error messages are program-generated and are written on the file OUTPUT. The error messages were designed to minimize the system-generated error messages.

# REFERENCES

1. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series 55, Issued June 1964, Fourth Printing, December 1965.

```
PROGRAM GAMINT     74/74   OPT=1                        FTN 4.2+P380       75/03/17. 09.57.05.

           PROGRAM GAMINT(INPUT,OUTPUT,INDAT,OUTA,TAPE5=INDAT,                   PAGE     1
          1  TAPE6=OUTA)
           COMMON / APLE / COT, CIT, COX
           INTEGER  COT , CIT
           CIT = 5
           COT = 6
           P = 0.0
        10 U = 0.0
           WPITE (COT , 20)
        20 FORMAT (1H1, 8X )
           DO 100  K = 1, 30
           U = U + 1.E-1
           X = U * SCRT (P + 1.0 )
           A = P + 1.0
           COX = 0.0
           COX = CONGAM (A)
           REF = 0.0
           REF =  COX - GAMMA (A,X)
           REF = REF  / COX
           WPITE ( COT , 50 ) U , P , REF
        50 FORMAT ( 8X, 4HU =  , 3X , F10.6 , 8X, 4HP =    , 3X , F10.6 ,
          1           8X, 9HI(U,P) =     , E19.12  )
       100 CONTINUE
           P = P + 5.E-1
           IF ( P . GE. 10.5 )  CALL EXIT
           GO TO 10
           END
```

```
      PROGRAM GAMINT     74/74   OPT=1                                  FTN 4.2+P380      75/03/17. 09.57.05.

     SYMBOLIC REFERENCE MAP (R=1)                                                                  PAGE     2

ENTRY POINTS
10214  GAMINT

VARIABLES     SN  TYPE            RELOCATION
10315  A           REAL                                1  CIT        INTEGER            ABLE
    0  COT         INTEGER            ABLE             2  COX        REAL               ABLE
10313  K           INTEGER                         10311  P          REAL
10316  REF         REAL                            10312  U          REAL
10314  X           REAL

FILE NAMES        MODE
 4102  INDAT                       0  INPUT               6143  OUTA                  2041  OUTPUT
 4102  TAPE5                    6143  TAPE6

EXTERNALS           TYPE   ARGS
       CONGAM    REAL       1                          EXIT                0
       GAMMA     REAL       2                          SQRT      REAL      1 LIBRARY

STATEMENT LABELS
10220  10                         10266  20      FMT                     10276  50      FMT
    0  100

COMMON BLOCKS   LENGTH
       ABLE          3

STATISTICS
  PROGRAM LENGTH               1138       75
  BUFFER LENGTH              102048     4228
  CM LABELED COMMON LENGTH       38        3
```

```
      FUNCTION  GAMMA (A,X)
      COMMON / ABLE / COT, CIT ,COX
      INTEGER COT,CIT
      GAMMA = 0.0
      EULERS = 5.77215664901533E-1
      IF (A.EQ.0.0.AND.X.EQ.0.0) GO TO 500
      UPDATE = 0.0
      IF (A.LT.0.0) GO TO 40
      IF (A.EQ.0.0) GO TO 30
C     THE VALUE OF A IS GREATER THAN ZERO
      IF (A.LE.1.E-10) GO TO 30
      CRP = ABS (X)
    1 IF (CRP.LE.1.E-10) GO TO 20
C     TEST TO DETERMINE RANGE OF  X WITH RESPECT TO A
      CRP = ABS (A)
      IF (X.LT.SQRT (CRP + 1.0 ))   GO TO 10
C     THE VALUE OF X IS GREATER  THAN THE SQROOT OF (ABS(A) +1 )
C     USE CONTINUED FRACTION TO APPROXIMATE INCOMPLETE GAMMA
C     INTEGRATING FROM X  TO INFINITY
      GAMMA = CONTFR (A,X)
      GO TO 500
C     VALUE OF X IS LESS THAN THE SQROOT OF (ABS(A) + 1)
C     USE SERIES EXPANSION INTEGRATING  X TO INFINITY
   10 GAMMA = EXP ( A * ALOG (X) ) * SMLGAM ( A, X )
      GAMMA = COX - GAMMA
      GO TO 500
   20 GAMMA = COX
      GO TO 500
   30 GAMMA = - (EULERS + ALOG (X)+ SMLGAM (0.0 , X))
      GO TO 500
   40 UPDATE = A + 1.0
      NP = 1
   50 IF (UPDATE.GE.0.0)  GO TO 60
      CRP = ABS (UPDATE)
      IF (CRP.LE.1.E-10)  GO TO 70
      UPDATE = UPDATE + 1.0
      NP = NP + 1
      GO TO 50
   60 IF (UPDATE.NE.0.0)  GO TO 80
   70 NS = NP
      GAMMA = GAMNEG (-NS, X )
      GO TO 500
   80 IF (UPDATE.LE.1.E-10)  GO TO 70
      GO TO 1
  500 RETURN
      END
```

```
        FUNCTION GAMMA      74/74   OPT=1                                FTN 4.2+P380      75/03/17. 09.57.05.
                                                                                                PAGE     2
      SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
    4  GAMMA

VARIABLES      SN  TYPE            RELOCATION
    0  A            REAL                  F.P.          1  CIT       INTEGER              ABLE
    0  COT          INTEGER             ABLE            2  COX       REAL                 ABLE
  134  CRP          REAL                              132  EULERS    REAL
  131  GAMMA        REAL                              135  NP        INTEGER
  136  NS           INTEGER                           133  UPDATE    REAL
    0  X            REAL                  F.P.

EXTERNALS          TYPE    ARGS
       ALOG        REAL       1 LIBRARY                  CONTFR    REAL       2
       EXP         REAL       1 LIBRARY                  GAMNEG    REAL       2
       SMLGAM      REAL       2                          SQRT      REAL       1 LIBRARY

INLINE FUNCTIONS   TYPE    ARGS
       ABS         REAL       1  INTRIN

STATEMENT LABELS
   21  1                            35  10                                   51  20
   53  30                           65  40                                   70  50
  100  60                          101  70                                  107  80
  112  500

COMMON BLOCKS   LENGTH
       ABLE          3

STATISTICS
  PROGRAM LENGTH              137B       95
  CM LABELED COMMON LENGTH      3B        3
```

```
FUNCTION SMLGAM     74/74   OPT=1                          FTN 4.2+P380       75/03/17. 09.57.06.

          FUNCTION  SMLGAM  (A ,X )                                                   PAGE     1
          COMMON / ABLE / COT, CIT, COX
          INTEGER  COT , CIT
C         SET INITIAL VALUES FOR SERIES EXPANSION I.E. N = 0  (6.5.29)
          SMLGAM = 0.0
          IF (A.NE.0.0)  SMLGAM = 1.0 / A
          ANUM = X
          PNEXT = 0.0
          FACTN = 1.0
          SIGN = 1.0
          EPSLON = 0.0
          DENOM = 0.0
          DO 10 K = 1, 30
          PN = K
          SIGN = - SIGN
          DENOM =  A+ PN
          FACTN = FACTN * PN
          TERM  = 0.0
          TERM  =  ( SIGN  * ANUM ) / (DENOM * FACTN )
          EPSLON = ABS (TERM)
          IF (EPSLON.LE.1.E-15)  GO TO  100
          SMLGAM = SMLGAM + TERM
          ANUM = ANUM * X
       10 CONTINUE
          WRITE (COT , 20)  EPSLON , A , X
       20 FORMAT(1H1, 8X, 35HSERIES FOR SMLGAM DID NOT CONVERGE  /
         1   8X,  9HEPSLON =  , E19.12 ,4X,4HA = ,E19.12,4X,4HX = ,E19.12 )
          CALL EXIT
      100 RETURN
          END
         FUNCTION SMLGAM     74/74   OPT=1                              FTN 4.2+P380       75/03/17. 09.57.06.
                                                                                                   PAGE     2
        SYMBOLIC REFERENCE MAP (R 1)

    ENTRY POINTS
        4  SMLGAM

    VARIABLES      SN  TYPE            RELOCATION
        0  A           REAL                F.P.     102  ANUM       REAL
        1  CIT         INTEGER            ABLE        0  COT        INTEGER            ABLE
        2  COX         REAL               ABLE      107  DENOM      REAL
      106  EPSLON      REAL                         104  FACTN      REAL
      110  K           INTEGER                      111  PN         REAL
      103  PNEXT       REAL                         105  SIGN       REAL
      101  SMLGAM      REAL                         112  TERM       REAL
        0  X           REAL                F.P.

    EXTERNALS          TYPE    ARGS
           EXIT                  0

    INLINE FUNCTIONS   TYPE    ARGS
           ABS        REAL       1  INTRIN

    STATEMENT LABELS
        0  10                          64  20  *   FMT                        54  100

    COMMON BLOCKS   LENGTH
           ABLE          3

    STATISTICS
      PROGRAM LENGTH              117B       79
      CM LABELED COMMON LENGTH      3B        3
```

```
FUNCTION CONTFR     74/74   OPT=1                          FTN 4.2+P380       75/03/17. 09.57.06.

                                                                                  PAGE     1
            FUNCTION  CONTFR  ( A , X )
            COMMON / ABLE /  COT , CIT ,COX
            INTEGER  COT , CIT
      C     SET INITIAL VALUES FOR  CONTINUED FRACTION   (6.5.31)
            ANUM = 25.0
            DENOM = X
            BNUM = 0.0
            DO  10  K = 1,30
            BNUM = ANUM / DENOM
            DENOM = BNUM + 1.0
            BNUM  = ANUM - A
            BNUM  = BNUM  / DENOM
            DENOM = BNUM + X
            ANUM = ANUM - 1.0
            IF (ANUM.LE.0.0)  GO TO 100
         10 CONTINUE
            WRITE (COT, 20)  ANUM , A , X
         20 FORMAT (1H1 ,9X ,37HCONTINUED FRACTION -CONTFR- DIVERGES
           1   9X,  8HANUM =  ,E19.12, 4X, 4HA =  ,E19.12,4X,4HX = , E19.12)
            CALL FXIT
        100 CONTFR = FXP (ALOG (X) * A - X ) / DENOM
            RETURN
            END
         FUNCTION CONTFR     74/74   OPT=1                                  FTN 4.2+P380       75/03/17. 09.57.06.
                                                                                                     PAGE     2

       SYMBOLIC REFERENCE MAP (R=1)

   ENTRY POINTS
      4  CONTFR

   VARIABLES      SN  TYPE           RELOCATION          64  ANUM      REAL                 ABLE
       0  A            REAL               F.P.            1  CIT       INTEGER              ABLE
      66  BNUM         REAL                               0  COT       INTEGER
      63  CONTFR       REAL                ABLE          65  DENOM     REAL                  F.P.
       2  COX          REAL                               0  X         REAL
      67  K            INTEGER

   EXTERNALS           TYPE   ARGS                            EXIT                0
         ALOG         REAL      1 LIBRARY
         EXP          REAL      1 LIBRARY

   STATEMENT LABELS                            46  20      FMT                         31  100
      0  10

   COMMON BLOCKS    LENGTH
         ABLE            3

   STATISTICS
     PROGRAM LENGTH                  70B      56
     CM LABELED COMMON LENGTH         3B       3
```

```
FUNCTION CONGAM     74/74   OPT=1                          FTN 4.2+P380       75/03/17. 09.57.07.      PAGE     1

          FUNCTION CONGAM  (ALF )
          COMMON / ABLE / COT, CIT, COX
          INTEGER  COT , CIT
          DIMENSION   BC (8)
          DATA((BC (K) , K = 1,8 )  =   -0.577191652 , 0.988205891 ,
         1   -0.897056937  , 0.918206857 ,  - 0.756704078  , 0.482199394,
         2     -0.193527818  ,   0.035868343    )
          PI = 3.14159265358979
          KSW = 1
          CONGAM = 0.0
          IF ( ALF . GT. 1. E-10  )  GO TO 1
          IF ( ALF . LT. 1. E-10  )  GO TO 5
          CONGAM = 1.0
          GO TO 100
        5 NCP = ALF
          SAVE = ALF
          ALF1 = ABS (NCP)
          COMP = 0.0
          COMP = ALF + ALF1
          IF ( ABS (COMP) .GT. 1.E-10 )    GO TO 15
          WRITE ( COT , 10 )
       10 FORMAT ( 8X,  28HALPHA IS A NEGATIVE INTEGER   )
          GO TO 100
       15 KSW = 2
          ALF = 1.0 - ALF
          IF ( ALF . GT. 1.E-10  )  GO TO 1
          WRITE ( COT , 18 )
       18 FORMAT ( 8X, 30HBAD EQUATION IN TERMS OF ALPHA   )
          CALL EXIT
       19 ALF = SAVE
          CONGAM = PI / ( CONGAM * SIN ( PI * ALF) )
          GO TO 100
        1 MARK = 1
          FB = 1.0
          N = ALF
          COMP = 0.0
          COMP = ALF + 1.E-10
          N = COMP
          WRITE ( COT , 8 )   N , ALF
        8 FORMAT ( 8X, I4 , 8X, E19.12 )
          KN = N - 1
          IF (KN.GE.1)  GO TO 20
          IF ( ALF .LT. 1.0 )  GO TO 9
          X = ALF - 1.0
          GO TO 40
        9 MARK = 2
          X = ALF
          GO TO 40
       20 DO 30  K = 1, KN
          FK = K
          FA = 0.0
          FA = ALF - FK
          FB = FB * FA
       30 CONTINUE
          X = FA - 1.0
          TEST = ABS (X)
          IF (TEST.GE.1.E-9)  GO TO 40
          FOFX = 1.0
          GO TO 60
       40 JB = 5
          FOFX = 0.0
       50 FOFX = FOFX + BC(JB)
          FOFX = FOFX * X
          JB = JB - 1
          IF (JB.GT.0)  GO TO 50
          FOFX = FOFX + 1.0
       60 CONGAM = FOFX * FB
          IF (MARK .EQ. 2 ) CONGAM = CONGAM / X
          IF ( KSW .EQ. 2 )  GO TO 19
      100 RETURN
          END
```

```
            FUNCTION CONGAM      74/74    OPT=1                               FTN 4.2+P380        75/03/17. 09.57.07.

CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM

     17    I                   BASIC EXTERNAL OR INTRINSIC FUNCTION CALLED WITH WRONG TYPE ARGUMENT.

                                                                                                          PAGE      3
     FUNCTION CONGAM     74/74    OPT=1                          FTN 4.2+P380      75/03/17. 09.57.07.

    SYMBOLIC REFERENCE MAP (R=1)                                                        PAGE     4

ENTRY POINTS
    4  CONGAM

VARIABLES      SN  TYPE             RELOCATION
    0  ALF         REAL                   F.P.          172  ALF1      REAL
  207  BC          RFAL        ARRAY                      1  CIT       INTEGER           ABLE
  173  COMP        REAL                                 165  CONGAM    REAL
    0  COT         INTEGER              ABLE              2  COX       REAL              ABLE
  203  FA          REAL                                 175  FB        REAL
  202  FK          REAL                                 205  FOFX      REAL
  206  JB          INTEGER                              201  K         INTEGER
  '77  KN          INTFGER                              167  KSW       INTEGER
  174  MARK        INTEGEP                              176  N         INTEGER
  170  NCP         INTEGER                              166  PI        REAL
  171  SAVE        REAL                                 204  TEST      PEAL
  200  X           REAL

EXTERNALS           TYPE   ARGS
      EXIT                   0                                SIN       REAL       1 LIBRARY

INLINE FUNCTIONS   TYPE   ARGS
      ABS         REAL      1  INTRIN

STATEMENT LABELS
   45  1                                  16  5                                156  8       FMT
   64  9                                 134  10      FMT                       30  15
  144  18      FMT                        37  19                                67  20
    0  30                                107  40                               111  50
  120  60                                130  100

COMMON BLOCKS   LENGTH
      ABLE           3

STATISTICS
  PROGRAM LENGTH               217B     143
  CM LABELED COMMON LENGTH      3B        3
```

```
FUNCTION GAMNEG     74/74   OPT=1                          FTN 4.2+P380       75/03/17. 09.57.08.

              FUNCTION GAMNEG(MINUSN,X)                                                 PAGE     1
              EULERS = 0.577215664901533
              N=-MINUSN
              NLESS1 = N - 1
              FACTJ=1.0
              XTERM=X
              J=1
              SUM=1./XTERM
              SIGN=1.
            1 SIGN=-SIGN
              XTERM=XTERM*X
              SUM=SUM+SIGN*FACTJ/XTERM
              J=J+1
              IF (J-NLESS1) 2,2,3
            2 FACTJ = FACTJ*FLOAT(J)
              GO TO 1
            3 FACTN=FACTJ*FLOAT (N)
              SIGN=-SIGN
              E1X=-(EULERS +ALOG(X)+SMLGAM ( 0.0 , X ) )
              GAMNEG=SIGN/FACTN*(E1X-SUM/EXP(X))
              RETURN
              END
```

```
        FUNCTION GAMNEG      74/74    OPT=1                          FTN 4.2+P380      75/03/17. 09.57.03.
                                                                                         PAGE     2
 CARD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM
     2    I                   CONSTANT TOO LONG.  HIGH ORDER DIGITS RETAINED, BUT SOME PRECISION LOST.
```

```
        FUNCTION GAMNEG     74/74   OPT=1                         FTN 4.2+P390      75/03/17. 09.57.08.

      SYMBOLIC REFERENCE MAP (R=1)                                                          PAGE     3

ENTRY POINTS
    4  GAMNEG

VARIABLES       SN  TYPE            RELOCATION
   64  EULERS     REAL                                 75  E1X       REAL
   67  FACTJ      REAL                                 74  FACTN     REAL
   63  GAMNEG     REAL                                 71  J         INTEGER
    0  MINUSN     INTEGER               F.P.           65  N         INTEGER
   66  NLESS1     INTEGER                              73  SIGN      REAL
   72  SUM        REAL                                  0  X         REAL               F.P.
   70  XTERM      REAL

EXTERNALS          TYPE   ARGS
       ALOG       REAL       1 LIBRARY                        EXP       REAL       1 LIBRARY
       SMLGAM     REAL       2

INLINE FUNCTIONS   TYPE   ARGS
       FLOAT      REAL       1  INTRIN

STATEMENT LABELS
   17  1                                  0  2          INACTIVE                32  3

STATISTICS
  PROGRAM LENGTH                76B       62
```

```
   1          .100000000000E+01
U =        .100000        P =        0.000000          I(U,P) =    .951625819640E-01
   1          .100000000000E+01
U =        .200000        P =        0.000000          I(U,P) =    .181269246922E+00
   1          .100000000000E+01
U =        .300000        P =        0.000000          I(U,P) =    .259181779318E+00
   1          .100000000000E+01
U =        .400000        P =        0.000000          I(U,P) =    .329679953964E+00
   1          .100000000000E+01
U =        .500000        P =        0.000000          I(U,P) =    .393469340287E+00
   1          .100000000000E+01
U =        .600000        P =        0.000000          I(U,P) =    .451188363906E+00
   1          .100000000000E+01
U =        .700000        P =        0.000000          I(U,P) =    .503414696209E+00
   1          .100000000000E+01
U =        .800000        P =        0.000000          I(U,P) =    .550671035883E+00
   1          .100000000000E+01
U =        .900000        P =        0.000000          I(U,P) =    .593430340259E+00
   1          .100000000000E+01
U =       1.000000        P =        0.000000          I(U,P) =    .632120558829E+00
   1          .100000000000E+01
U =       1.100000        P =        0.000000          I(U,P) =    .667128916302E+00
   1          .100000000000E+01
U =       1.200000        P =        0.000000          I(U,P) =    .698805788088E+00
   1          .100000000000E+01
U =       1.300000        P =        0.000000          I(U,P) =    .727468206966E+00
   1          .100000000000E+01
U =       1.400000        P =        0.000000          I(U,P) =    .753403036058E+00
   1          .100000000000E+01
U =       1.500000        P =        0.000000          I(U,P) =    .776869839852E+00
   1          .100000000000E+01
U =       1.600000        P =        0.000000          I(U,P) =    .798103482005E+00
   1          .100000000000E+01
U =       1.700000        P =        0.000000          I(U,P) =    .817316475947E+00
   1          .100000000000E+01
U =       1.800000        P =        0.000000          I(U,P) =    .834701111778E+00
   1          .100000000000E+01
U =       1.900000        P =        0.000000          I(U,P) =    .850431380777E+00
   1          .100000000000E+01
U =       2.000000        P =        0.000000          I(U,P) =    .864664716763E+00
   1          .100000000000E+01
U =       2.100000        P =        0.000000          I(U,P) =    .877543571747E+00
   1          .100000000000E+01
U =       2.200000        P =        0.000000          I(U,P) =    .889196841638E+00
   1          .100000000000E+01
U =       2.300000        P =        0.000000          I(U,P) =    .899741156277E+00
   1          .100000000000E+01
U =       2.400000        P =        0.000000          I(U,P) =    .909282046711E+00
   1          .100000000000E+01
U =       2.500000        P =        0.000000          I(U,P) =    .917915001376E+00
   1          .100000000000E+01
U =       2.600000        P =        0.000000          I(U,P) =    .925726421786E+00
   1          .100000000000E+01
U =       2.700000        P =        0.000000          I(U,P) =    .932794487260E+00
   1          .100000000000E+01
U =       2.800000        P =        0.000000          I(U,P) =    .939189937375E+00
   1          .100000000000E+01
U =       2.900000        P =        0.000000          I(U,P) =    .944976779944E+00
   1          .100000000000E+01
U =       3.000000        P =        0.000000          I(U,P) =    .950212931632E+00
```

```
   1          .150000000000E+01
U =       .100000         P =          .500000          I(U,P) =    .299738381032E-01
   1          .150000000000E+01
U =       .200000         P =          .500000          I(U,P) =    .788948769845E-01
   1          .150000000000E+01
U =       .300000         P =          .500000          I(U,P) =    .135021406834E+00
   1          .150000000000E+01
U =       .400000         P =          .500000          I(U,P) =    .193859394269E+00
   1          .150000000000E+01
U =       .500000         P =          .500000          I(U,P) =    .252924188106E+00
   1          .150000000000E+01
U =       .600000         P =          .500000          I(U,P) =    .310717546602E+00
   1          .150000000000E+01
U =       .700000         P =          .500000          I(U,P) =    .366316687408E+00
   1          .150000000000E+01
U =       .800000         P =          .500000          I(U,P) =    .419154172685E+00
   1          .150000000000E+01
U =       .900000         P =          .500000          I(U,P) =    .468945427412E+00
   1          .150000000000E+01
U =      1.000000         P =          .500000          I(U,P) =    .515511346563E+00
   1          .150000000000E+01
U =      1.100000         P =          .500000          I(U,P) =    .558926751971E+00
   1          .150000000000E+01
U =      1.200000         P =          .500000          I(U,P) =    .598934831444E+00
   1          .150000000000E+01
U =      1.300000         P =          .500000          I(U,P) =    .635932093263E+00
   1          .150000000000E+01
U =      1.400000         P =          .500000          I(U,P) =    .669950017549E+00
   1          .150000000000E+01
U =      1.500000         P =          .500000          I(U,P) =    .701142473508E+00
   1          .150000000000E+01
U =      1.600000         P =          .500000          I(U,P) =    .729675903407E+00
   1          .150000000000E+01
U =      1.700000         P =          .500000          I(U,P) =    .755722500574E+00
   1          .150000000000E+01
U =      1.800000         P =          .500000          I(U,P) =    .779455223888E+00
   1          .150000000000E+01
U =      1.900000         P =          .500000          I(U,P) =    .801044238722E+00
   1          .150000000000E+01
U =      2.000000         P =          .500000          I(U,P) =    .820654425914E+00
   1          .150000000000E+01
U =      2.100000         P =          .500000          I(U,P) =    .838443692722E+00
   1          .150000000000E+01
U =      2.200000         P =          .500000          I(U,P) =    .854561886119E+00
   1          .150000000000E+01
U =      2.300000         P =          .500000          I(U,P) =    .869150157405E+00
   1          .150000000000E+01
U =      2.400000         P =          .500000          I(U,P) =    .882340663108E+00
   1          .150000000000E+01
U =      2.500000         P =          .500000          I(U,P) =    .894256514228E+00
   1          .150000000000E+01
U =      2.600000         P =          .500000          I(U,P) =    .905011906389E+00
   1          .150000000000E+01
U =      2.700000         P =          .500000          I(U,P) =    .914712379103E+00
   1          .150000000000E+01
U =      2.800000         P =          .500000          I(U,P) =    .923455164457E+00
   1          .150000000000E+01
U =      2.900000         P =          .500000          I(U,P) =    .931329594803E+00
   1          .150000000000E+01
U =      3.000000         P =          .500000          I(U,P) =    .938417546306E+00
```

```
    2              .200000000000E+01
U =        .100000           P =        1.000000          I(U,P) =   .910535957631E-02
    2              .200000000000E+01
U =        .200000           P =        1.000000          I(U,P) =   .332005779085E-01
    2              .200000000000E+01
U =        .300000           P =        1.000000          I(U,P) =   .681736779589E-01
    2              .200000000000E+01
U =        .400000           P =        1.000000          I(U,P) =   .110736534404E+00
    2              .200000000000E+01
U =        .500000           P =        1.000000          I(U,P) =   .158279093328E+00
    2              .200000000000E+01
U =        .600000           P =        1.000000          I(U,P) =   .208747713966E+00
    2              .200000000000E+01
U =        .700000           P =        1.000000          I(U,P) =   .260543805562E+00
    2              .200000000000E+01
U =        .800000           P =        1.000000          I(U,P) =   .312439522262E+00
    2              .200000000000E+01
U =        .900000           P =        1.000000          I(U,P) =   .363507779758E+00
    2              .200000000000E+01
U =       1.000000           P =        1.000000          I(U,P) =   .413064282489E+00
    2              .200000000000E+01
U =       1.100000           P =        1.000000          I(U,P) =   .460619610671E+00
    2              .200000000000E+01
U =       1.200000           P =        1.000000          I(U,P) =   .505839722081E+00
    2              .200000000000E+01
U =       1.300000           P =        1.000000          I(U,P) =   .548513482448E+00
    2              .200000000000E+01
U =       1.400000           P =        1.000000          I(U,P) =   .588526057571E+00
    2              .200000000000E+01
U =       1.500000           P =        1.000000          I(U,P) =   .625837185802E+00
    2              .200000000000E+01
U =       1.600000           P =        1.000000          I(U,P) =   .660463506433E+00
    2              .200000000000E+01
U =       1.700000           P =        1.000000          I(U,P) =   .692464252103E+00
    2              .200000000000E+01
U =       1.800000           P =        1.000000          I(U,P) =   .721929725290E+00
    2              .200000000000E+01
U =       1.900000           P =        1.000000          I(U,P) =   .748972073405E+00
    2              .200000000000E+01
U =       2.000000           P =        1.000000          I(U,P) =   .773717956634E+00
    2              .200000000000E+01
U =       2.100000           P =        1.000000          I(U,P) =   .796302769744E+00
    2              .200000000000E+01
U =       2.200000           P =        1.000000          I(U,P) =   .816866135509E+00
    2              .200000000000E+01
U =       2.300000           P =        1.000000          I(U,P) =   .835548434854E+00
    2              .200000000000E+01
U =       2.400000           P =        1.000000          I(U,P) =   .852488178648E+00
    2              .200000000000E+01
U =       2.500000           P =        1.000000          I(U,P) =   .867820059517E+00
    2              .200000000000E+01
U =       2.600000           P =        1.000000          I(U,P) =   .881673550010E+00
    2              .200000000000E+01
U =       2.700000           P =        1.000000          I(U,P) =   .894171936921E+00
    2              .200000000000E+01
U =       2.800000           P =        1.000000          I(U,P) =   .905431701082E+00
    2              .200000000000E+01
U =       2.900000           P =        1.000000          I(U,P) =   .915562168327E+00
    2              .200000000000E+01
U =       3.000000           P =        1.000000          I(U,P) =   .924665370879E+00
```

```
  2         .250000000000E+01
U =       .100000        P =        1.500000        I(U,P) =   .267330444585E-02
  2         .250000000000E+01
U =       .200000        P =        1.500000        I(U,P) =   .135309254562E-01
  2         .250000000000E+01
U =       .300000        P =        1.500000        I(U,P) =   .334022749229E-01
  2         .250000000000E+01
U =       .400000        P =        1.500000        I(U,P) =   .614994481181E-01
  2         .250000000000E+01
U =       .500000        P =        1.500000        I(U,P) =   .964792350408E-01
  2         .250000000000E+01
U =       .600000        P =        1.500000        I(U,P) =   .136843705324E+00
  2         .250000000000E+01
U =       .700000        P =        1.500000        I(U,P) =   .181130047716E+00
  2         .250000000000E+01
U =       .800000        P =        1.500000        I(U,P) =   .228002727155E+00
  2         .250000000000E+01
U =       .900000        P =        1.500000        I(U,P) =   .276294094660E+00
  2         .250000000000E+01
U =      1.000000        P =        1.500000        I(U,P) =   .325016350121E+00
  2         .250000000000E+01
U =      1.100000        P =        1.500000        I(U,P) =   .373357472183E+00
  2         .250000000000E+01
U =      1.200000        P =        1.500000        I(U,P) =   .420668556938E+00
  2         .250000000000E+01
U =      1.300000        P =        1.500000        I(U,P) =   .466446514439E+00
  2         .250000000000E+01
U =      1.400000        P =        1.500000        I(U,P) =   .510316000298E+00
  2         .250000000000E+01
U =      1.500000        P =        1.500000        I(U,P) =   .552010604088E+00
  2         .250000000000E+01
U =      1.600000        P =        1.500000        I(U,P) =   .591355422768E+00
  2         .250000000000E+01
U =      1.700000        P =        1.500000        I(U,P) =   .628251046803E+00
  2         .250000000000E+01
U =      1.800000        P =        1.500000        I(U,P) =   .662659268538E+00
  2         .250000000000E+01
U =      1.900000        P =        1.500000        I(U,P) =   .694590569814E+00
  2         .250000000000E+01
U =      2.000000        P =        1.500000        I(U,P) =   .724093339562E+00
  2         .250000000000E+01
U =      2.100000        P =        1.500000        I(U,P) =   .751244710060E+00
  2         .250000000000E+01
U =      2.200000        P =        1.500000        I(U,P) =   .776142867613E+00
  2         .250000000000E+01
U =      2.300000        P =        1.500000        I(U,P) =   .798900679612E+00
  2         .250000000000E+01
U =      2.400000        P =        1.500000        I(U,P) =   .819640478332E+00
  2         .250000000000E+01
U =      2.500000        P =        1.500000        I(U,P) =   .838489847845E+00
  2         .250000000000E+01
U =      2.600000        P =        1.500000        I(U,P) =   .855578270898E+00
  2         .250000000000E+01
U =      2.700000        P =        1.500000        I(U,P) =   .871034505400E+00
  2         .250000000000E+01
U =      2.800000        P =        1.500000        I(U,P) =   .884984573906E+00
  2         .250000000000E+01
U =      2.900000        P =        1.500000        I(U,P) =   .897550263227E+00
  2         .250000000000E+01
U =      3.000000        P =        1.500000        I(U,P) =   .908848044463E+00
```

```
   3          .300000000000E+01
U =       .100000        P =        2.000000        I(U,P) =   .760958139402E-03
   3          .300000000000E+01
U =       .200000        P =        2.000000        I(U,P) =   .535529738963E-02
   3          .300000000000E+01
U =       .300000        P =        2.000000        I(U,P) =   .159186794226E-01
   3          .300000000000E+01
U =       .400000        P =        2.000000        I(U,P) =   .332739084163E-01
   3          .300000000000E+01
U =       .500000        P =        2.000000        I(U,P) =   .573798362723E-01
   3          .300000000000E+01
U =       .600000        P =        2.000000        I(U,P) =   .876571174908E-01
   3          .300000000000E+01
U =       .700000        P =        2.000000        I(U,P) =   .123220788784E+00
   3          .300000000000E+01
U =       .800000        P =        2.000000        I(U,P) =   .163042885807E+00
   3          .300000000000E+01
U =       .900000        P =        2.000000        I(U,P) =   .206062994147E+00
   3          .300000000000E+01
U =      1.000000        P =        2.000000        I(U,P) =   .251260465927E+00
   3          .300000000000E+01
U =      1.100000        P =        2.000000        I(U,P) =   .297698778525E+00
   3          .300000000000E+01
U =      1.200000        P =        2.000000        I(U,P) =   .344549977546E+00
   3          .300000000000E+01
U =      1.300000        P =        2.000000        I(U,P) =   .391105180196E+00
   3          .300000000000E+01
U =      1.400000        P =        2.000000        I(U,P) =   .436775596703E+00
   3          .300000000000E+01
U =      1.500000        P =        2.000000        I(U,P) =   .481087359999E+00
   3          .300000000000E+01
U =      1.600000        P =        2.000000        I(U,P) =   .523672561332E+00
   3          .300000000000E+01
U =      1.700000        P =        2.000000        I(U,P) =   .564258211228E+00
   3          .300000000000E+01
U =      1.800000        P =        2.000000        I(U,P) =   .602654333554E+00
   3          .300000000000E+01
U =      1.900000        P =        2.000000        I(U,P) =   .638742017683E+00
   3          .300000000000E+01
U =      2.000000        P =        2.000000        I(U,P) =   .672461970338E+00
   3          .300000000000E+01
U =      2.100000        P =        2.000000        I(U,P) =   .703803901366E+00
   3          .300000000000E+01
U =      2.200000        P =        2.000000        I(U,P) =   .732796928359E+00
   3          .300000000000E+01
U =      2.300000        P =        2.000000        I(U,P) =   .759501079600E+00
   3          .300000000000E+01
U =      2.400000        P =        2.000000        I(U,P) =   .783999902269E+00
   3          .300000000000E+01
U =      2.500000        P =        2.000000        I(U,P) =   .806394134801E+00
   3          .300000000000E+01
U =      2.600000        P =        2.000000        I(U,P) =   .826796372210E+00
   3          .300000000000E+01
U =      2.700000        P =        2.000000        I(U,P) =   .845326636151E+00
   3          .300000000000E+01
U =      2.800000        P =        2.000000        I(U,P) =   .862108753689E+00
   3          .300000000000E+01
U =      2.900000        P =        2.000000        I(U,P) =   .877267447376E+00
   3          .300000000000E+01
U =      3.000000        P =        2.000000        I(U,P) =   .890926042158E+00
```

```
   3           .350000000000E+01
U =        .100000          P =       2.500000          I(U,P) =   .210631087928E-03
   3           .350000000000E+01
U =        .200000          P =       2.500000          I(U,P) =   .206380282799E-02
   3           .350000000000E+01
U =        .300000          P =       2.500000          I(U,P) =   .739668242440E-02
   3           .350000000000E+01
U =        .400000          P =       2.500000          I(U,P) =   .175750353806E-01
   3           .350000000000E+01
U =        .500000          P =       2.500000          I(U,P) =   .333576129694E-01
   3           .350000000000E+01
U =        .600000          P =       2.500000          I(U,P) =   .549539458907E-01
   3           .350000000000E+01
U =        .700000          P =       2.500000          I(U,P) =   .821398303100E-01
   3           .350000000000E+01
U =        .800000          P =       2.500000          I(U,P) =   .114381038707E+00
   3           .350000000000E+01
U =        .900000          P =       2.500000          I(U,P) =   .150945111460E+00
   3           .350000000000E+01
U =       1.000000          P =       2.500000          I(U,P) =   .190993899399E+00
   3           .350000000000E+01
U =       1.100000          P =       2.500000          I(U,P) =   .233655561592E+00
   3           .350000000000E+01
U =       1.200000          P =       2.500000          I(U,P) =   .278077611147E+00
   3           .350000000000E+01
U =       1.300000          P =       2.500000          I(U,P) =   .323463213192E+00
   3           .350000000000E+01
U =       1.400000          P =       2.500000          I(U,P) =   .369094925600E+00
   3           .350000000000E+01
U =       1.500000          P =       2.500000          I(U,P) =   .414347174868E+00
   3           .350000000000E+01
U =       1.600000          P =       2.500000          I(U,P) =   .458691225432E+00
   3           .350000000000E+01
U =       1.700000          P =       2.500000          I(U,P) =   .501694339172E+00
   3           .350000000000E+01
U =       1.800000          P =       2.500000          I(U,P) =   .543014980674E+00
   3           .350000000000E+01
U =       1.900000          P =       2.500000          I(U,P) =   .582395475985E+00
   3           .350000000000E+01
U =       2.000000          P =       2.500000          I(U,P) =   .619653209499E+00
   3           .350000000000E+01
U =       2.100000          P =       2.500000          I(U,P) =   .654671170730E+00
   3           .350000000000E+01
U =       2.200000          P =       2.500000          I(U,P) =   .687388438736E+00
   3           .350000000000E+01
U =       2.300000          P =       2.500000          I(U,P) =   .717791012611E+00
   3           .350000000000E+01
U =       2.400000          P =       2.500000          I(U,P) =   .745903255988E+00
   3           .350000000000E+01
U =       2.500000          P =       2.500000          I(U,P) =   .771780115975E+00
   3           .350000000000E+01
U =       2.600000          P =       2.500000          I(U,P) =   .795500196554E+00
   3           .350000000000E+01
U =       2.700000          P =       2.500000          I(U,P) =   .817159708082E+00
   3           .350000000000E+01
U =       2.800000          P =       2.500000          I(U,P) =   .836867273540E+00
   3           .350000000000E+01
U =       2.900000          P =       2.500000          I(U,P) =   .854739544760E+00
   3           .350000000000E+01
U =       3.000000          P =       2.500000          I(U,P) =   .870897564846E+00
```

```
    4            .400000000000E+01
U =        .100000          P =        3.000000        I(U,P) =    .568402407595E-04
    4            .400000000000E+01
U =        .200000          P =        3.000000        I(U,P) =    .776251376209E-03
    4            .400000000000E+01
U =        .300000          P =        3.000000        I(U,P) =    .335806885325E-02
    4            .400000000000E+01
U =        .400000          P =        3.000000        I(U,P) =    .907985780015E-02
    4            .400000000000E+01
U =        .500000          P =        3.000000        I(U,P) =    .189881568762E-01
    4            .400000000000E+01
U =        .600000          P =        3.000000        I(U,P) =    .337689681857E-01
    4            .400000000000E+01
U =        .700000          P =        3.000000        I(U,P) =    .537252503681E-01
    4            .400000000000E+01
U =        .800000          P =        3.000000        I(U,P) =    .788134872297E-01
    4            .400000000000E+01
U =        .900000          P =        3.000000        I(U,P) =    .108708394709E+00
    4            .400000000000E+01
U =       1.000000          P =        3.000000        I(U,P) =    .142876539501E+00
    4            .400000000000E+01
U =       1.100000          P =        3.000000        I(U,P) =    .180647578297E+00
    4            .400000000000E+01
U =       1.200000          P =        3.000000        I(U,P) =    .221277088964E+00
    4            .400000000000E+01
U =       1.300000          P =        3.000000        I(U,P) =    .263998355615E+00
    4            .400000000000E+01
U =       1.400000          P =        3.000000        I(U,P) =    .308062567409E+00
    4            .400000000000E+01
U =       1.500000          P =        3.000000        I(U,P) =    .352768111218E+00
    4            .400000000000E+01
U =       1.600000          P =        3.000000        I(U,P) =    .397480275594E+00
    4            .400000000000E+01
U =       1.700000          P =        3.000000        I(U,P) =    .441642944717E+00
    4            .400000000000E+01
U =       1.800000          P =        3.000000        I(U,P) =    .484783889534E+00
    4            .400000000000E+01
U =       1.900000          P =        3.000000        I(U,P) =    .526515156740E+00
    4            .400000000000E+01
U =       2.000000          P =        3.000000        I(U,P) =    .566529879633E+00
    4            .400000000000E+01
U =       2.100000          P =        3.000000        I(U,P) =    .604596630398E+00
    4            .400000000000E+01
U =       2.200000          P =        3.000000        I(U,P) =    .640552227211E+00
    4            .400000000000E+01
U =       2.300000          P =        3.000000        I(U,P) =    .674293716979E+00
    4            .400000000000E+01
U =       2.400000          P =        3.000000        I(U,P) =    .705770083503E+00
    4            .400000000000E+01
U =       2.500000          P =        3.000000        I(U,P) =    .734974084703E+00
    4            .400000000000E+01
U =       2.600000          P =        3.000000        I(U,P) =    .761934501277E+00
    4            .400000000000E+01
U =       2.700000          P =        3.000000        I(U,P) =    .786708981566E+00
    4            .400000000000E+01
U =       2.800000          P =        3.000000        I(U,P) =    .809377590658E+00
    4            .400000000000E+01
U =       2.900000          P =        3.000000        I(U,P) =    .830037113334E+00
    4            .400000000000E+01
U =       3.000000          P =        3.000000        I(U,P) =    .848796117223E+00
```

```
   4           .450000000000E+01
U =        .100000        P =        3.500000          I(U,P) =    .149871972877E-04
   4           .450000000000E+01
U =        .200000        P =        3.500000          I(U,P) =    .285540751958E-03
   4           .450000000000E+01
U =        .300000        P =        3.500000          I(U,P) =    .149235389445E-02
   4           .450000000000E+01
U =        .400000        P =        3.500000          I(U,P) =    .459607067587E-02
   4           .450000000000E+01
U =        .500000        P =        3.500000          I(U,P) =    .105995474937E-01
   4           .450000000000E+01
U =        .600000        P =        3.500000          I(U,P) =    .203676699138E-01
   4           .450000000000E+01
U =        .700000        P =        3.500000          I(U,P) =    .345216302672E-01
   4           .450000000000E+01
U =        .800000        P =        3.500000          I(U,P) =    .533966254760E-01
   4           .450000000000E+01
U =        .900000        P =        3.500000          I(U,P) =    .770454230236E-01
   4           .450000000000E+01
U =       1.000000        P =        3.500000          I(U,P) =    .105271027820E+00
   4           .450000000000E+01
U =       1.100000        P =        3.500000          I(U,P) =    .137674864065E+00
   4           .450000000000E+01
U =       1.200000        P =        3.500000          I(U,P) =    .173710571114E+00
   4           .450000000000E+01
U =       1.300000        P =        3.500000          I(U,P) =    .212736197655E+00
   4           .450000000000E+01
U =       1.400000        P =        3.500000          I(U,P) =    .254061864958E+00
   4           .450000000000E+01
U =       1.500000        P =        3.500000          I(U,P) =    .296989153312E+00
   4           .450000000000E+01
U =       1.600000        P =        3.500000          I(U,P) =    .340842638548E+00
   4           .450000000000E+01
U =       1.700000        P =        3.500000          I(U,P) =    .384993256643E+00
   4           .450000000000E+01
U =       1.800000        P =        3.500000          I(U,P) =    .428874291372E+00
   4           .450000000000E+01
U =       1.900000        P =        3.500000          I(U,P) =    .471990997232E+00
   4           .450000000000E+01
U =       2.000000        P =        3.500000          I(U,P) =    .513924982460E+00
   4           .450000000000E+01
U =       2.100000        P =        3.500000          I(U,P) =    .554334467916E+00
   4           .450000000000E+01
U =       2.200000        P =        3.500000          I(U,P) =    .592951453869E+00
   4           .450000000000E+01
U =       2.300000        P =        3.500000          I(U,P) =    .629576702316E+00
   4           .450000000000E+01
U =       2.400000        P =        3.500000          I(U,P) =    .664073301112E+00
   4           .450000000000E+01
U =       2.500000        P =        3.500000          I(U,P) =    .696359433648E+00
   4           .450000000000E+01
U =       2.600000        P =        3.500000          I(U,P) =    .726400843851E+00
   4           .450000000000E+01
U =       2.700000        P =        3.500000          I(U,P) =    .754203366510E+00
   4           .450000000000E+01
U =       2.800000        P =        3.500000          I(U,P) =    .779805789752E+00
   4           .450000000000E+01
U =       2.900000        P =        3.500000          I(U,P) =    .803273230576E+00
   4           .450000000000E+01
U =       3.000000        P =        3.500000          I(U,P) =    .824691134833E+00
```

```
     5              .500000000000E+01
U =          .100000         P =          4.000000          I(U,P) =   .386844019810E-05
     5              .500000000000E+01
U =          .200000         P =          4.000000          I(U,P) =   .102902325551E-03
     5              .500000000000E+01
U =          .300000         P =          4.000000          I(U,P) =   .650256682216E-03
     5              .500000000000E+01
U =          .400000         P =          4.000000          I(U,P) =   .228277528583E-02
     5              .500000000000E+01
U =          .500000         P =          4.000000          I(U,P) =   .581027876542E-02
     5              .500000000000E+01
U =          .600000         P =          4.000000          I(U,P) =   .120726954337E-01
     5              .500000000000E+01
U =          .700000         P =          4.000000          I(U,P) =   .218160022451E-01
     5              .500000000000E+01
U =          .800000         P =          4.000000          I(U,P) =   .356062979064E-01
     5              .500000000000E+01
U =          .900000         P =          4.000000          I(U,P) =   .537843137798E-01
     5              .500000000000E+01
U =         1.000000         P =          4.000000          I(U,P) =   .764545246626E-01
     5              .500000000000E+01
U =         1.100000         P =          4.000000          I(U,P) =   .103500033500E+00
     5              .500000000000E+01
U =         1.200000         P =          4.000000          I(U,P) =   .134614223283E+00
     5              .500000000000E+01
U =         1.300000         P =          4.000000          I(U,P) =   .169341354500E+00
     5              .500000000000E+01
U =         1.400000         P =          4.000000          I(U,P) =   .207119987525E+00
     5              .500000000000E+01
U =         1.500000         P =          4.000000          I(U,P) =   .247324853932E+00
     5              .500000000000E+01
U =         1.600000         P =          4.000000          I(U,P) =   .289304351590E+00
     5              .500000000000E+01
U =         1.700000         P =          4.000000          I(U,P) =   .332412099248E+00
     5              .500000000000E+01
U =         1.800000         P =          4.000000          I(U,P) =   .376031941197E+00
     5              .500000000000E+01
U =         1.900000         P =          4.000000          I(U,P) =   .419596467617E+00
     5              .500000000000E+01
U =         2.000000         P =          4.000000          I(U,P) =   .462599555919E+00
     5              .500000000000E+01
U =         2.100000         P =          4.000000          I(U,P) =   .504603592355E+00
     5              .500000000000E+01
U =         2.200000         P =          4.000000          I(U,P) =   .545242948257E+00
     5              .500000000000E+01
U =         2.300000         P =          4.000000          I(U,P) =   .584222502152E+00
     5              .500000000000E+01
U =         2.400000         P =          4.000000          I(U,P) =   .621315551034E+00
     5              .500000000000E+01
U =         2.500000         P =          4.000000          I(U,P) =   .656358367091E+00
     5              .500000000000E+01
U =         2.600000         P =          4.000000          I(U,P) =   .689244149514E+00
     5              .500000000000E+01
U =         2.700000         P =          4.000000          I(U,P) =   .719916208315E+00
     5              .500000000000E+01
U =         2.800000         P =          4.000000          I(U,P) =   .748360907630E+00
     5              .500000000000E+01
U =         2.900000         P =          4.000000          I(U,P) =   .774600695374E+00
     5              .500000000000E+01
U =         3.000000         P =          4.000000          I(U,P) =   .798687457631E+00
```

```
    5          .550000000000E+01
U =        .100000          P =        4.500000        I(U,P) =    .979052949357E-06
    5          .550000000000E+01
U =        .200000          P =        4.500000        I(U,P) =    .363856540980E-04
    5          .550000000000E+01
U =        .300000          P =        4.500000        I(U,P) =    .278188121101E-03
    5          .550000000000E+01
U =        .400000          P =        4.500000        I(U,P) =    .111396815083E-02
    5          .550000000000E+01
U =        .500000          P =        4.500000        I(U,P) =    .313134873584E-02
    5          .550000000000E+01
U =        .600000          P =        4.500000        I(U,P) =    .704016180112E-02
    5          .550000000000E+01
U =        .700000          P =        4.500000        I(U,P) =    .135726268703E-01
    5          .550000000000E+01
U =        .800000          P =        4.500000        I(U,P) =    .233901186024E-01
    5          .550000000000E+01
U =        .900000          P =        4.500000        I(U,P) =    .370120156238E-01
    5          .550000000000E+01
U =       1.000000          P =        4.500000        I(U,P) =    .547720726608E-01
    5          .550000000000E+01
U =       1.100000          P =        4.500000        I(U,P) =    .768013704951E-01
    5          .550000000000E+01
U =       1.200000          P =        4.500000        I(U,P) =    .103032846785E+00
    5          .550000000000E+01
U =       1.300000          P =        4.500000        I(U,P) =    .133222705238E+00
    5          .550000000000E+01
U =       1.400000          P =        4.500000        I(U,P) =    .166981191453E+00
    5          .550000000000E+01
U =       1.500000          P =        4.500000        I(U,P) =    .203808734319E+00
    5          .550000000000E+01
U =       1.600000          P =        4.500000        I(U,P) =    .243132846445E+00
    5          .550000000000E+01
U =       1.700000          P =        4.500000        I(U,P) =    .284342866362E+00
    5          .550000000000E+01
U =       1.800000          P =        4.500000        I(U,P) =    .326820590280E+00
    5          .550000000000E+01
U =       1.900000          P =        4.500000        I(U,P) =    .369965719890E+00
    5          .550000000000E+01
U =       2.000000          P =        4.500000        I(U,P) =    .413215739353E+00
    5          .550000000000E+01
U =       2.100000          P =        4.500000        I(U,P) =    .456060334149E+00
    5          .550000000000E+01
U =       2.200000          P =        4.500000        I(U,P) =    .498050798708E+00
    5          .550000000000E+01
U =       2.300000          P =        4.500000        I(U,P) =    .538805077975E+00
    5          .550000000000E+01
U =       2.400000          P =        4.500000        I(U,P) =    .578009180646E+00
    5          .550000000000E+01
U =       2.500000          P =        4.500000        I(U,P) =    .615415717092E+00
    5          .550000000000E+01
U =       2.600000          P =        4.500000        I(U,P) =    .650840277338E+00
    5          .550000000000E+01
U =       2.700000          P =        4.500000        I(U,P) =    .684156293751E+00
    5          .550000000000E+01
U =       2.800000          P =        4.500000        I(U,P) =    .715288944621E+00
    5          .550000000000E+01
U =       2.900000          P =        4.500000        I(U,P) =    .744208559995E+00
    5          .550000000000E+01
U =       3.000000          P =        4.500000        I(U,P) =    .770923897889E+00
```

```
       6            .600000000000E+01
U  =          .100000          P  =       5.000000       I(U,P)  =    .243299064095E-06
       6            .600000000000E+01
U  =          .200000          P  =       5.000000       I(U,P)  =    .126402101121E-04
       6            .600000000000E+01
U  =          .300000          P  =       5.000000       I(U,P)  =    .116994872936E-03
       6            .600000000000E+01
U  =          .400000          P  =       5.000000       I(U,P)  =    .534702654689E-03
       6            .600000000000E+01
U  =          .500000          P  =       5.000000       I(U,P)  =    .166092958849E-02
       6            .600000000000E+01
U  =          .600000          P  =       5.000000       I(U,P)  =    .404297832832E-02
       6            .600000000000E+01
U  =          .700000          P  =       5.000000       I(U,P)  =    .832046426602E-02
       6            .600000000000E+01
U  =          .800000          P  =       5.000000       I(U,P)  =    .151490951164E-01
       6            .600000000000E+01
U  =          .900000          P  =       5.000000       I(U,P)  =    .251264546651E-01
       6            .600000000000E+01
U  =         1.000000          P  =       5.000000       I(U,P)  =    .387318188487E-01
       6            .600000000000E+01
U  =         1.100000          P  =       5.000000       I(U,P)  =    .562858174785E-01
       6            .600000000000E+01
U  =         1.200000          P  =       5.000000       I(U,P)  =    .779309358306E-01
       6            .600000000000E+01
U  =         1.300000          P  =       5.000000       I(U,P)  =    .103630685515E+00
       6            .600000000000E+01
U  =         1.400000          P  =       5.000000       I(U,P)  =    .133183571672E+00
       6            .600000000000E+01
U  =         1.500000          P  =       5.000000       I(U,P)  =    .166247391751E+00
       6            .600000000000E+01
U  =         1.600000          P  =       5.000000       I(U,P)  =    .202369555856E+00
       6            .600000000000E+01
U  =         1.700000          P  =       5.000000       I(U,P)  =    .241019709478E+00
       6            .600000000000E+01
U  =         1.800000          P  =       5.000000       I(U,P)  =    .281621730698E+00
       6            .600000000000E+01
U  =         1.900000          P  =       5.000000       I(U,P)  =    .323583003038E+00
       6            .600000000000E+01
U  =         2.000000          P  =       5.000000       I(U,P)  =    .366319630794E+00
       6            .600000000000E+01
U  =         2.100000          P  =       5.000000       I(U,P)  =    .409276911943E+00
       6            .600000000000E+01
U  =         2.200000          P  =       5.000000       I(U,P)  =    .451944893718E+00
       6            .600000000000E+01
U  =         2.300000          P  =       5.000000       I(U,P)  =    .493869208648E+00
       6            .600000000000E+01
U  =         2.400000          P  =       5.000000       I(U,P)  =    .534657638058E+00
       6            .600000000000E+01
U  =         2.500000          P  =       5.000000       I(U,P)  =    .573982995647E+00
       6            .600000000000E+01
U  =         2.600000          P  =       5.000000       I(U,P)  =    .611582987813E+00
       6            .600000000000E+01
U  =         2.700000          P  =       5.000000       I(U,P)  =    .647257710899E+00
       6            .600000000000E+01
U  =         2.800000          P  =       5.000000       I(U,P)  =    .680865407109E+00
       6            .600000000000E+01
U  =         2.900000          P  =       5.000000       I(U,P)  =    .712317036081E+00
       6            .600000000000E+01
U  =         3.000000          P  =       5.000000       I(U,P)  =    .741570140259E+00
```

```
   6           .650000000000E+01
U =          .100000        P =       5.500000        I(U,P) =    .594387125411E-07
   6           .650000000000E+01
U =          .200000        P =       5.500000        I(U,P) =    .431914085561E-05
   6           .650000000000E+01
U =          .300000        P =       5.500000        I(U,P) =    .484215690064E-04
   6           .650000000000E+01
U =          .400000        P =       5.500000        I(U,P) =    .252709108807E-03
   6           .650000000000E+01
U =          .500000        P =       5.500000        I(U,P) =    .867891594198E-03
   6           .650000000000E+01
U =          .600000        P =       5.500000        I(U,P) =    .228844395138E-02
   6           .650000000000E+01
U =          .700000        P =       5.500000        I(U,P) =    .503010315961E-02
   6           .650000000000E+01
U =          .800000        P =       5.500000        I(U,P) =    .968081719702E-02
   6           .650000000000E+01
U =          .900000        P =       5.500000        I(U,P) =    .168389772730E-01
   6           .650000000000E+01
U =         1.000000        P =       5.500000        I(U,P) =    .270518485724E-01
   6           .650000000000E+01
U =         1.100000        P =       5.500000        I(U,P) =    .407637429051E-01
   6           .650000000000E+01
U =         1.200000        P =       5.500000        I(U,P) =    .582788609896E-01
   6           .650000000000E+01
U =         1.300000        P =       5.500000        I(U,P) =    .797418033012E-01
   6           .650000000000E+01
U =         1.400000        P =       5.500000        I(U,P) =    .105133281349E+00
   6           .650000000000E+01
U =         1.500000        P =       5.500000        I(U,P) =    .134279649810E+00
   6           .650000000000E+01
U =         1.600000        P =       5.500000        I(U,P) =    .166872381584E+00
   6           .650000000000E+01
U =         1.700000        P =       5.500000        I(U,P) =    .202493996452E+00
   6           .650000000000E+01
U =         1.800000        P =       5.500000        I(U,P) =    .240647152284E+00
   6           .650000000000E+01
U =         1.900000        P =       5.500000        I(U,P) =    .280784142559E+00
   6           .650000000000E+01
U =         2.000000        P =       5.500000        I(U,P) =    .322334688795E+00
   6           .650000000000E+01
U =         2.100000        P =       5.500000        I(U,P) =    .364730566889E+00
   6           .650000000000E+01
U =         2.200000        P =       5.500000        I(U,P) =    .407426196488E+00
   6           .650000000000E+01
U =         2.300000        P =       5.500000        I(U,P) =    .449914816457E+00
   6           .650000000000E+01
U =         2.400000        P =       5.500000        I(U,P) =    .491740254226E+00
   6           .650000000000E+01
U =         2.500000        P =       5.500000        I(U,P) =    .532504573858E+00
   6           .650000000000E+01
U =         2.600000        P =       5.500000        I(U,P) =    .571872068340E+00
   6           .650000000000E+01
U =         2.700000        P =       5.500000        I(U,P) =    .609570161251E+00
   6           .650000000000E+01
U =         2.800000        P =       5.500000        I(U,P) =    .645387818958E+00
   6           .650000000000E+01
U =         2.900000        P =       5.500000        I(U,P) =    .679172063379E+00
   6           .650000000000E+01
U =         3.000000        P =       5.500000        I(U,P) =    .710823131816E+00
```

```
    7            .700000000000E+01
U =         .100000          P =          6.000000          I(U,P) =   .142905902490E-07
    7            .700000000000E+01
U =         .200000          P =          6.000000          I(U,P) =   .145311601045E-05
    7            .700000000000E+01
U =         .300000          P =          6.000000          I(U,P) =   .197410519730E-04
    7            .700000000000E+01
U =         .400000          P =          6.000000          I(U,P) =   .117703397746E-03
    7            .700000000000E+01
U =         .500000          P =          6.000000          I(U,P) =   .447136299676E-03
    7            .700000000000E+01
U =         .600000          P =          6.000000          I(U,P) =   .127773376040E-02
    7            .700000000000E+01
U =         .700000          P =          6.000000          I(U,P) =   .300101915953E-02
    7            .700000000000E+01
U =         .800000          P =          6.000000          I(U,P) =   .610803285805E-02
    7            .700000000000E+01
U =         .900000          P =          6.000000          I(U,P) =   .111471976893E-01
    7            .700000000000E+01
U =        1.000000          P =          6.000000          I(U,P) =   .186720848133E-01
    7            .700000000000E+01
U =        1.100000          P =          6.000000          I(U,P) =   .291887301672E-01
    7            .700000000000E+01
U =        1.200000          P =          6.000000          I(U,P) =   .431101164283E-01
    7            .700000000000E+01
U =        1.300000          P =          6.000000          I(U,P) =   .607226382537E-01
    7            .700000000000E+01
U =        1.400000          P =          6.000000          I(U,P) =   .821666246459E-01
    7            .700000000000E+01
U =        1.500000          P =          6.000000          I(U,P) =   .107430750342E+00
    7            .700000000000E+01
U =        1.600000          P =          6.000000          I(U,P) =   .136358599779E+00
    7            .700000000000E+01
U =        1.700000          P =          6.000000          I(U,P) =   .168664745054E+00
    7            .700000000000E+01
U =        1.800000          P =          6.000000          I(U,P) =   .203957358485E+00
    7            .700000000000E+01
U =        1.900000          P =          6.000000          I(U,P) =   .241764457555E+00
    7            .700000000000E+01
U =        2.000000          P =          6.000000          I(U,P) =   .281561230260E+00
    7            .700000000000E+01
U =        2.100000          P =          6.000000          I(U,P) =   .322796386640E+00
    7            .700000000000E+01
U =        2.200000          P =          6.000000          I(U,P) =   .364916029694E+00
    7            .700000000000E+01
U =        2.300000          P =          6.000000          I(U,P) =   .407384066895E+00
    7            .700000000000E+01
U =        2.400000          P =          6.000000          I(U,P) =   .449698649167E+00
    7            .700000000000E+01
U =        2.500000          P =          6.000000          I(U,P) =   .491404505111E+00
    7            .700000000000E+01
U =        2.600000          P =          6.000000          I(U,P) =   .532101327827E+00
    7            .700000000000E+01
U =        2.700000          P =          6.000000          I(U,P) =   .571448573653E+00
    7            .700000000000E+01
U =        2.800000          P =          6.000000          I(U,P) =   .609167156717E+00
    7            .700000000000E+01
U =        2.900000          P =          6.000000          I(U,P) =   .645038583806E+00
    7            .700000000000E+01
U =        3.000000          P =          6.000000          I(U,P) =   .678902084864E+00
```

```
   7             .750000000000E+01
U =          .100000          P =        6.500000        I(U,P) =   .338471493279E-08
   7             .750000000000E+01
U =          .200000          P =        6.500000        I(U,P) =   .481785734928E-06
   7             .750000000000E+01
U =          .300000          P =        6.500000        I(U,P) =   .793470753991E-05
   7             .750000000000E+01
U =          .400000          P =        6.500000        I(U,P) =   .540711301228E-04
   7             .750000000000E+01
U =          .500000          P =        6.500000        I(U,P) =   .227301595918E-03
   7             .750000000000E+01
U =          .600000          P =        6.500000        I(U,P) =   .704220872141E-03
   7             .750000000000E+01
U =          .700000          P =        6.500000        I(U,P) =   .176811243196E-02
   7             .750000000000E+01
U =          .800000          P =        6.500000        I(U,P) =   .380733532425E-02
   7             .750000000000E+01
U =          .900000          P =        6.500000        I(U,P) =   .729335873351E-02
   7             .750000000000E+01
U =         1.000000          P =        6.500000        I(U,P) =   .127433076815E-01
   7             .750000000000E+01
U =         1.100000          P =        6.500000        I(U,P) =   .206744510149E-01
   7             .750000000000E+C1
U =         1.200000          P =        6.500000        I(U,P) =   .315577200064E-01
   7             .750000000000E+01
U =         1.300000          P =        6.500000        I(U,P) =   .457777184633E-01
   7             .750000000000E+01
U =         1.400000          P =        6.500000        I(U,P) =   .636018741201E-01
   7             .750000000000E+01
U =         1.500000          P =        6.500000        I(U,P) =   .851620147143E-01
   7             .750000000000E+01
U =         1.600000          P =        6.500000        I(U,P) =   .110448103854E+00
   7             .750000000000E+01
U =         1.700000          P =        6.500000        I(U,P) =   .139313016119E+00
   7             .750000000000E+01
U =         1.800000          P =        6.500000        I(U,P) =   .171486254611E+00
   7             .750000000000E+01
U =         1.900000          P =        6.500000        I(U,P) =   .206594088593E+00
   7             .750000000000E+01
U =         2.000000          P =        6.500000        I(U,P) =   .244183541203E+00
   7             .750000000000E+01
U =         2.100000          P =        6.500000        I(U,P) =   .283747878082E+00
   7             .750000000000E+01
U =         2.200000          P =        6.500000        I(U,P) =   .324751632678E+00
   7             .750000000000E+01
U =         2.300000          P =        6.500000        I(U,P) =   .366653664893E+00
   7             .750000000000E+01
U =         2.400000          P =        6.500000        I(U,P) =   .408927219666E+00
   7             .750000000000E+01
U =         2.500000          P =        6.500000        I(U,P) =   .451076385081E+00
   7             .750000000000E+01
U =         2.600000          P =        6.500000        I(U,P) =   .492648718310E+00
   7             .750000000000E+01
U =         2.700000          P =        6.500000        I(U,P) =   .533244099671E+00
   7             .750000000000E+01
U =         2.800000          P =        6.500000        I(U,P) =   .572520088707E+00
   7             .750000000000E+01
U =         2.900000          P =        6.500000        I(U,P) =   .610194197240E+00
   7             .750000000000E+01
U =         3.000000          P =        6.500000        I(U,P) =   .646043572857E+00
```

```
     8           .800000000000E+01
U =         .100000          P =         7.000000         I(U,P) =    .790365332265E-09
     8           .800000000000E+01
U =         .200000          P =         7.000000         I(U,P) =    .157546033041E-06
     8           .800000000000E+01
U =         .300000          P =         7.000000         I(U,P) =    .314667657164E-05
     8           .800000000000E+01
U =         .400000          P =         7.000000         I(U,P) =    .245168027577E-04
     8           .800000000000E+01
U =         .500000          P =         7.000000         I(U,P) =    .114090387496E-03
     8           .800000000000E+01
U =         .600000          P =         7.000000         I(U,P) =    .383375315436E-03
     8           .800000000000E+01
U =         .700000          P =         7.000000         I(U,P) =    .102934515699E-02
     8           .800000000000E+01
U =         .800000          P =         7.000000         I(U,P) =    .234592846191E-02
     8           .800000000000E+01
U =         .900000          P =         7.000000         I(U,P) =    .471875691890E-02
     8           .800000000000E+01
U =        1.000000          P =         7.000000         I(U,P) =    .860348352718E-02
     8           .800000000000E+01
U =        1.100000          P =         7.000000         I(U,P) =    .144916267066E-01
     8           .800000000000E+01
U =        1.200000          P =         7.000000         I(U,P) =    .228698123597E-01
     8           .800000000000E+01
U =        1.300000          P =         7.000000         I(U,P) =    .341785736742E-01
     8           .800000000000E+01
U =        1.400000          P =         7.000000         I(U,P) =    .487759757958E-01
     8           .800000000000E+01
U =        1.500000          P =         7.000000         I(U,P) =    .669097897290E-01
     8           .800000000000E+01
U =        1.600000          P =         7.000000         I(U,P) =    .887002082724E-01
     8           .800000000000E+01
U =        1.700000          P =         7.000000         I(U,P) =    .114133505735E+00
     8           .800000000000E+01
U =        1.800000          P =         7.000000         I(U,P) =    .143065783032E+00
     8           .800000000000E+01
U =        1.900000          P =         7.000000         I(U,P) =    .175235084615E+00
     8           .800000000000E+01
U =        2.000000          P =         7.000000         I(U,P) =    .210279714613E+00
     8           .800000000000E+01
U =        2.100000          P =         7.000000         I(U,P) =    .247760457313E+00
     8           .800000000000E+01
U =        2.200000          P =         7.000000         I(U,P) =    .287184539303E+00
     8           .800000000000E+01
U =        2.300000          P =         7.000000         I(U,P) =    .328029471018E+00
     8           .800000000000E+01
U =        2.400000          P =         7.000000         I(U,P) =    .369765296527E+00
     8           .800000000000E+01
U =        2.500000          P =         7.000000         I(U,P) =    .411874199154E+00
     8           .800000000000E+01
U =        2.600000          P =         7.000000         I(U,P) =    .453866811178E+00
     8           .800000000000E+01
U =        2.700000          P =         7.000000         I(U,P) =    .495294928284E+00
     8           .800000000000E+01
U =        2.800000          P =         7.000000         I(U,P) =    .535760617712E+00
     8           .800000000000E+01
U =        2.900000          P =         7.000000         I(U,P) =    .574921927511E+00
     8           .800000000000E+01
U =        3.000000          P =         7.000000         I(U,P) =    .612495555479E+00
```

```
     8          .850000000000E+01
U =        .100000           P =        7.500000        I(U,P) =    .182095837449E-09
     8          .850000000000E+01
U =        .200000           P =        7.500000        I(U,P) =    .508480136276E-07
     8          .850000000000E+01
U =        .300000           P =        7.500000        I(U,P) =    .123205984309E-05
     8          .850000000000E+01
U =        .400000           P =        7.500000        I(U,P) =    .109791059282E-04
     8          .850000000000E+01
U =        .500000           P =        7.500000        I(U,P) =    .565779660635E-04
     8          .850000000000E+01
U =        .600000           P =        7.500000        I(U,P) =    .206271268076E-03
     8          .850000000000E+01
U =        .700000           P =        7.500000        I(U,P) =    .592461039310E-03
     8          .850000000000E+01
U =        .800000           P =        7.500000        I(U,P) =    .142956832437E-02
     8          .850000000000E+01
U =        .900000           P =        7.500000        I(U,P) =    .302046357801E-02
     8          .850000000000E+01
U =       1.000000           P =        7.500000        I(U,P) =    .574860417956E-02
     8          .850000000000E+01
U =       1.100000           P =        7.500000        I(U,P) =    .100566114550E-01
     8          .850000000000F+01
U =       1.200000           P =        7.500000        I(U,P) =    .164142012203E-01
     8          .850000000000E+01
U =       1.300000           P =        7.500000        I(U,P) =    .252815524307E-01
     8          .850000000000E+01
U =       1.400000           P =        7.500000        I(U,P) =    .370716926947E-01
     8          .850000000000E+01
U =       1.500000           P =        7.500000        I(U,P) =    .521175984915E-01
     8          .850000000000E+01
U =       1.600000           P =        7.500000        I(U,P) =    .706468114080E-01
     8          .850000000000E+01
U =       1.700000           P =        7.500000        I(U,P) =    .927654945747E-01
     8          .850000000000F+01
U =       1.800000           P =        7.500000        I(U,P) =    .118452407963E+00
     8          .850000000000E+01
U =       1.900000           P =        7.500000        I(U,P) =    .147562157439E+00
     8          .850000000000E+01
U =       2.000000           P =        7.500000        I(U,P) =    .179836273753E+00
     8          .850000000000E+01
U =       2.100000           P =        7.500000        I(U,P) =    .214920213144E+00
     8          .850000000000E+01
U =       2.200000           P =        7.500000        I(U,P) =    .252384212764E+00
     8          .850000000000E+01
U =       2.300000           P =        7.500000        I(U,P) =    .291745996539E+00
     8          .850000000000E+01
U =       2.400000           P =        7.500000        I(U,P) =    .332493578848E+00
     8          .850000000000E+01
U =       2.500000           P =        7.500000        I(U,P) =    .374106738806E+00
     8          .850000000000E+01
U =       2.600000           P =        7.500000        I(U,P) =    .416076118792E+00
     8          .850000000000E+01
U =       2.700000           P =        7.500000        I(U,P) =    .457919269985E+00
     8          .850000000000E+01
U =       2.800000           P =        7.500000        I(U,P) =    .499193302756E+00
     8          .850000000000E+01
U =       2.900000           P =        7.500000        I(U,P) =    .539504080634E+00
     8          .850000000000E+01
U =       3.000000           P =        7.500000        I(U,P) =    .578512115414E+00
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```
     9           .900000000000E+01
U =        .100000        P =      8.000000      I(U,P) =    .414267618446E-10
     9           .900000000000E+01
U =        .200000        P =      8.000000      I(U,P) =    .162082581064E-07
     9           .900000000000E+01
U =        .300000        P =      8.000000      I(U,P) =    .476584330225E-06
     9           .900000000000E+01
U =        .400000        P =      8.000000      I(U,P) =    .485882870664E-05
     9           .900000000000E+01
U =        .500000        P =      8.000000      I(U,P) =    .277358192501E-04
     9           .900000000000E+01
U =        .600000        P =      8.000000      I(U,P) =    .109744644936E-03
     9           .900000000000E+01
U =        .700000        P =      8.000000      I(U,P) =    .337305843837E-03
     9           .900000000000E+01
U =        .800000        P =      8.000000      I(U,P) =    .861978256338E-03
     9           .900000000000E+01
U =        .900000        P =      8.000000      I(U,P) =    .191362749731E-02
     9           .900000000000E+01
U =       1.000000        P =      8.000000      I(U,P) =    .380299206168E-02
     9           .900000000000E+01
U =       1.100000        P =      8.000000      I(U,P) =    .691181223987E-02
     9           .900000000000E+01
U =       1.200000        P =      8.000000      I(U,P) =    .116714122388E-01
     9           .900000000000E+01
U =       1.300000        P =      8.000000      I(U,P) =    .185327565006E-01
     9           .900000000000E+01
U =       1.400000        P =      8.000000      I(U,P) =    .279321909591E-01
     9           .900000000000E+01
U =       1.500000        P =      8.000000      I(U,P) =    .402573124822E-01
     9           .900000000000E+01
U =       1.600000        P =      8.000000      I(U,P) =    .558168774858E-01
     9           .900000000000E+01
U =       1.700000        P =      8.000000      I(U,P) =    .748176447461E-01
     9           .900000000000E+01
U =       1.800000        P =      8.000000      I(U,P) =    .973498298869E-01
     9           .900000000000E+01
U =       1.900000        P =      8.000000      I(U,P) =    .123381659325E+00
     9           .900000000000E+01
U =       2.000000        P =      8.000000      I(U,P) =    .152762506016E+00
     9           .900000000000E+01
U =       2.100000        P =      8.000000      I(U,P) =    .185233350710E+00
     9           .900000000000E+01
U =       2.200000        P =      8.000000      I(U,P) =    .220442862409E+00
     9           .900000000000E+01
U =       2.300000        P =      8.000000      I(U,P) =    .257967208184E+00
     9           .900000000000E+01
U =       2.400000        P =      8.000000      I(U,P) =    .297331737576E+00
     9           .900000000000E+01
U =       2.500000        P =      8.000000      I(U,P) =    .338032880859E+00
     9           .900000000000E+01
U =       2.600000        P =      8.000000      I(U,P) =    .379558893902E+00
     9           .900000000000E+01
U =       2.700000        P =      8.000000      I(U,P) =    .421408422283E+00
     9           .900000000000E+01
U =       2.800000        P =      8.000000      I(U,P) =    .463106201245E+00
     9           .900000000000E+01
U =       2.900000        P =      8.000000      I(U,P) =    .504215525652E+00
     9           .900000000000E+01
U =       3.000000        P =      8.000000      I(U,P) =    .544347395678E+00
```

```
   9          .950000000000E+01
U =        .100000        P =       8.500000          I(U,P) =   .930990997548E-11
   9          .950000000000E+01
U =        .200000        P =       8.500000          I(U,P) =   .510564441166E-08
   9          .950000000000E+01
U =        .300000        P =       8.500000          I(U,P) =   .182230370527E-06
   9          .950000000000E+01
U =        .400000        P =       8.500000          I(U,P) =   .212613666528E-05
   9          .950000000000E+01
U =        .500000        P =       8.500000          I(U,P) =   .134478314618E-04
   9          .950000000000E+01
U =        .600000        P =       8.500000          I(U,P) =   .577655392964E-04
   9          .950000000000E+01
U =        .700000        P =       8.500000          I(U,P) =   .190043409319E-03
   9          .950000000000E+01
U =        .800000        P =       8.500000          I(U,P) =   .514490348248E-03
   9          .950000000000E+01
U =        .900000        P =       8.500000          I(U,P) =   .120048302901E-02
   9          .950000000000E+01
U =       1.000000        P =       8.500000          I(U,P) =   .249189172884E-02
   9          .950000000000E+01
U =       1.100000        P =       8.500000          I(U,P) =   .470663152170E-02
   9          .950000000000E+01
U =       1.200000        P =       8.500000          I(U,P) =   .822484344730E-02
   9          .950000000000E+01
U =       1.300000        P =       8.500000          I(U,P) =   .134679566469E-01
   9          .950000000000E+01
U =       1.400000        P =       8.500000          I(U,P) =   .208698780140E-01
   9          .950000000000E+01
U =       1.500000        P =       8.500000          I(U,P) =   .308450131107E-01
   9          .950000000000E+01
U =       1.600000        P =       8.500000          I(U,P) =   .437568782255E-01
   9          .950000000000E+01
U =       1.700000        P =       8.500000          I(U,P) =   .598904753377E-01
   9          .950000000000E+01
U =       1.800000        P =       8.500000          I(U,P) =   .794312002035E-01
   9          .950000000000E+01
U =       1.900000        P =       8.500000          I(U,P) =   .102451720734E+00
   9          .950000000000E+01
U =       2.000000        P =       8.500000          I(U,P) =   .128907291095E+00
   9          .950000000000E+01
U =       2.100000        P =       8.500000          I(U,P) =   .158639053029E+00
   9          .950000000000E+01
U =       2.200000        P =       8.500000          I(U,P) =   .191384196688E+00
   9          .950000000000E+01
U =       2.300000        P =       8.500000          I(U,P) =   .226791425522E+00
   9          .950000000000E+01
U =       2.400000        P =       8.500000          I(U,P) =   .264439980988E+00
   9          .950000000000E+01
U =       2.500000        P =       8.500000          I(U,P) =   .303860495071E+00
   9          .950000000000E+01
U =       2.600000        P =       8.500000          I(U,P) =   .344556101671E+00
   9          .950000000000E+01
U =       2.700000        P =       8.500000          I(U,P) =   .386022499102E+00
   9          .950000000000E+01
U =       2.800000        P =       8.500000          I(U,P) =   .427765966896E+00
   9          .950000000000E+01
U =       2.900000        P =       8.500000          I(U,P) =   .469318661017E+00
   9          .950000000000E+01
U =       3.000000        P =       8.500000          I(U,P) =   .510250812672E+00
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  10          .100000000000E+02
U =        .100000          P =        9.000000          I(U,P) =   .206857913123E-11
  10          .100000000000E+02
U =        .200000          P =        9.000000          I(U,P) =   .159019325914E-08
  10          .100000000000E+02
U =        .300000          P =        9.000000          I(U,P) =   .689125630505E-07
  10          .100000000000E+02
U =        .400000          P =        9.000000          I(U,P) =   .920363673546E-06
  10          .100000000000E+02
U =        .500000          P =        9.000000          I(U,P) =   .645186045992E-05
  10          .100000000000E+02
U =        .600000          P =        9.000000          I(U,P) =   .300946425492E-04
  10          .100000000000E+02
U =        .700000          P =        9.000000          I(U,P) =   .106006062549E-03
  10          .100000000000E+02
U =        .800000          P =        9.000000          I(U,P) =   .304103794899E-03
  10          .100000000000E+02
U =        .900000          P =        9.000000          I(U,P) =   .745991245749E-03
  10          .100000000000E+02
U =       1.000000          P =        9.000000          I(U,P) =   .161781194689E-02
  10          .100000000000E+02
U =       1.100000          P =        9.000000          I(U,P) =   .317632962698E-02
  10          .100000000000E+02
U =       1.200000          P =        9.000000          I(U,P) =   .574582622049E-02
  10          .100000000000E+02
U =       1.300000          P =        9.000000          I(U,P) =   .970522716690E-02
  10          .100000000000E+02
U =       1.400000          P =        9.000000          I(U,P) =   .154666621515E-01
  10          .100000000000E+02
U =       1.500000          P =        9.000000          I(U,P) =   .234480495215E-01
  10          .100000000000E+02
U =       1.600000          P =        9.000000          I(U,P) =   .340430507750E-01
  10          .100000000000E+02
U =       1.700000          P =        9.000000          I(U,P) =   .475918578779E-01
  10          .100000000000E+02
U =       1.800000          P =        9.000000          I(U,P) =   .643558602915E-01
  10          .100000000000E+02
U =       1.900000          P =        9.000000          I(U,P) =   .844934667048E-01
  10          .100000000000E+02
U =       2.000000          P =        9.000000          I(U,P) =   .108073418263E+00
  10          .100000000000F+02
U =       2.100000          P =        9.000000          I(U,P) =   .135020991277E+00
  10          .100000000000E+02
U =       2.200000          P =        9.000000          I(U,P) =   .165171670081E+00
  10          .100000000000E+02
U =       2.300000          P =        9.000000          I(U,P) =   .198256247792E+00
  10          .100000000000E+02
U =       2.400000          P =        9.000000          I(U,P) =   .233920912469E+00
  10          .100000000000E+02
U =       2.500000          P =        9.000000          I(U,P) =   .271745691527E+00
  10          .100000000000E+02
U =       2.600000          P =        9.000000          I(U,P) =   .311264627674E+00
  10          .100000000000E+02
U =       2.700000          P =        9.000000          I(U,P) =   .351986201967E+00
  10          .100000000000E+02
U =       2.800000          P =        9.000000          I(U,P) =   .393412756655E+00
  10          .100000000000E+02
U =       2.900000          P =        9.000000          I(U,P) =   .435057958383E+00
  10          .100000000000E+02
U =       3.000000          P =        9.000000          I(U,P) =   .476461643469E+00
```

```
   10              .105000000000E+02
U =          .100000          P =          9.500000          I(U,P) =     .460205193536E-12
   10              .105000000000E+02
U =          .200000          P =          9.500000          I(U,P) =     .489947597758E-09
   10              .105000000000E+02
U =          .300000          P =          9.500000          I(U,P) =     .257855928400E-07
   10              .105000000000E+02
U =          .400000          P =          9.500000          I(U,P) =     .394305012929E-06
   10              .105000000000E+02
U =          .500000          P =          9.500000          I(U,P) =     .306426474973E-05
   10              .105000000000E+02
U =          .600000          P =          9.500000          I(U,P) =     .155246877136E-04
   10              .105000000000E+02
U =          .700000          P =          9.500000          I(U,P) =     .585635347848E-04
   10              .105000000000E+02
U =          .800000          P =          9.500000          I(U,P) =     .178070158513E-03
   10              .105000000000E+02
U =          .900000          P =          9.500000          I(U,P) =     .459349017203E-03
   10              .105000000000E+02
U =         1.000000          P =          9.500000          I(U,P) =     .104103554753E-02
   10              .105000000000E+02
U =         1.100000          P =          9.500000          I(U,P) =     .212525522714E-02
   10              .105000000000E+02
U =         1.200000          P =          9.500000          I(U,P) =     .398057051097E-02
   10              .105000000000E+02
U =         1.300000          P =          9.500000          I(U,P) =     .693715125005E-02
   10              .105000000000E+02
U =         1.400000          P =          9.500000          I(U,P) =     .113723938641E-01
   10              .105000000000E+02
U =         1.500000          P =          9.500000          I(U,P) =     .176895041379E-01
   10              .105000000000E+02
U =         1.600000          P =          9.500000          I(U,P) =     .262910218696E-01
   10              .105000000000E+02
U =         1.700000          P =          9.500000          I(U,P) =     .375504242656E-01
   10              .105000000000E+02
U =         1.800000          P =          9.500000          I(U,P) =     .517849235822E-01
   10              .105000000000E+02
U =         1.900000          P =          9.500000          I(U,P) =     .692321793600E-01
   10              .105000000000E+02
U =         2.000000          P =          9.500000          I(U,P) =     .900329444843E-01
   10              .105000000000E+02
U =         2.100000          P =          9.500000          I(U,P) =     .114220818023E+00
   10              .105000000000E+02
U =         2.200000          P =          9.500000          I(U,P) =     .141719429456E+00
   10              .105000000000E+02
U =         2.300000          P =          9.500000          I(U,P) =     .172346636386E+00
   10              .105000000000E+02
U =         2.400000          P =          9.500000          I(U,P) =     .205824747571E+00
   10              .105000000000E+02
U =         2.500000          P =          9.500000          I(U,P) =     .241795412813E+00
   10              .105000000000E+02
U =         2.600000          P =          9.500000          I(U,P) =     .279837647117E+00
   10              .105000000000E+02
U =         2.700000          P =          9.500000          I(U,P) =     .319487453106E+00
   10              .105000000000E+02
U =         2.800000          P =          9.500000          I(U,P) =     .360257634872E+00
   10              .105000000000E+02
U =         2.900000          P =          9.500000          I(U,P) =     .401656616144E+00
   10              .105000000000E+02
U =         3.000000          P =          9.500000          I(U,P) =     .443205345213E+00
```

```
  11            .110000000000E+02
U =         .100000         P =      10.000000          I(U,P) =    .102659013957E-12
  11            .110000000000E+02
U =         .200000         P =      10.000000          I(U,P) =    .149397609831E-09
  11            .110000000000E+02
U =         .300000         P =      10.000000          I(U,P) =    .955091832071E-08
  11            .110000000000E+02
U =         .400000         P =      10.000000          I(U,P) =    .167259276307E-06
  11            .110000000000E+02
U =         .500000         P =      10.000000          I(U,P) =    .144128248299E-05
  11            .110000000000E+02
U =         .600000         P =      10.000000          I(U,P) =    .793295672222E-05
  11            .110000000000E+02
U =         .700000         P =      10.000000          I(U,P) =    .320553138681E-04
  11            .110000000000E+02
U =         .800000         P =      10.000000          I(U,P) =    .103332014156E-03
  11            .110000000000E+02
U =         .900000         P =      10.000000          I(U,P) =    .280365904229E-03
  11            .110000000000E+02
U =        1.000000         P =      10.000000          I(U,P) =    .664164957745E-03
  11            .110000000000E+02
U =        1.100000         P =      10.000000          I(U,P) =    .141005858395E-02
  11            .110000000000E+02
U =        1.200000         P =      10.000000          I(U,P) =    .273523484008E-02
  11            .110000000000E+02
U =        1.300000         P =      10.000000          I(U,P) =    .491951015795E-02
  11            .110000000000E+02
U =        1.400000         P =      10.000000          I(U,P) =    .829809250156E-02
  11            .110000000000E+02
U =        1.500000         P =      10.000000          I(U,P) =    .132464763288E-01
  11            .110000000000E+02
U =        1.600000         P =      10.000000          I(U,P) =    .201588866974E-01
  11            .110000000000E+02
U =        1.700000         P =      10.000000          I(U,P) =    .294226248135E-01
  11            .110000000000E+02
U =        1.800000         P =      10.000000          I(U,P) =    .413911392892E-01
  11            .110000000000E+02
U =        1.900000         P =      10.000000          I(U,P) =    .563586499588E-01
  11            .110000000000E+02
U =        2.000000         P =      10.000000          I(U,P) =    .745387640343E-01
  11            .110000000000E+02
U =        2.100000         P =      10.000000          I(U,P) =    .960488741491E-01
  11            .110000000000E+02
U =        2.200000         P =      10.000000          I(U,P) =    .120901353495E+00
  11            .110000000000E+02
U =        2.300000         P =      10.000000          I(U,P) =    .149001789678E+00
  11            .110000000000E+02
U =        2.400000         P =      10.000000          I(U,P) =    .180153821524E+00
  11            .110000000000E+02
U =        2.500000         P =      10.000000          I(U,P) =    .214069621828E+00
  11            .110000000000E+02
U =        2.600000         P =      10.000000          I(U,P) =    .250384729791E+00
  11            .110000000000E+02
U =        2.700000         P =      10.000000          I(U,P) =    .288675777595E+00
  11            .110000000000E+02
U =        2.800000         P =      10.000000          I(U,P) =    .328479653862E+00
  11            .110000000000E+02
U =        2.900000         P =      10.000000          I(U,P) =    .369312768885E+00
  11            .110000000000E+02
U =        3.000000         P =      10.000000          I(U,P) =    .410689293846E+00
```